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A new mathematical model towards the integration of cell formation with operator assignment and inter-cell layout problems in a dynamic environment

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ABSTRACT

This paper proposes a new mathematical model to solve the cell formation, operator assignment and inter-cell layout problems, simultaneously. The objectives of proposed model are minimization of inter–intra cell part trips, machine relocation cost and operator related issues. Since the objective function of the proposed model consists of none commensurable statements, the preferred solution is obtained by the LP-metric approach. In order to validate the proposed model, some numerical examples are generated randomly and solved by branch and bound technique. Moreover; a real case study is illustrated in order to verify its applicability in an automobile producer company. Moreover the sensitivity analysis of proposed model shows that considering the operator assignment problem has significant impact on the overall system efficiency.

1. Introduction

Cellular manufacturing system (CMS) is an application of group technology concept in which parts and machines should be assigned to production cells with respect to their similarities in production process, design, shape, etc. Designing of a CMS involves four main steps which each of them can be treated as a separate problem. The first one is cell formation (CF) problem which concerns with grouping parts and machines in order to minimize some objectives such as inter–intra cell part trips. The second problem is finding the optimal layout of machines and cells. Actually the overall system efficiency depends on the optimal layout of machines within cells and cells within the shop floor. The third decision is part scheduling problem within cells. The main aim of this problem is total process completion time reduction. The recently recognized decision is operator assignment problem. Operator-related issues such as training, hiring, firing and salary are very important to be analyzed because of economical limitations in industrial plants. Moreover, Dynamic Cellular Manufacturing System (DCMS) is a production system which deals with designing cellular manufacturing system over a production horizon while manufacturing parameters such as demand and processing time are different in each production period. Hence, an optimal cell design in a period may not be optimal for remaining periods in a dynamic environment and optimization of mentioned objectives in such environment is desirable.

Since, the cell formation problem is the first decision in designing a CMS, many researchers have tried to solve this problem optimally. Schaller [1] proposed a mathematical model for CF problem in presence of stochastic demands and five heuristic methods were implemented to solve the problem. Majazi-Delfard [2] proposed a nonlinear mathematical model for a dynamic CF problem based on number and average length of inter–intra cell movements. Since the proposed model is

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completely NP-hard a simulated annealing embedded in branch and cut algorithm was applied to solve the problem. Ameli and Arkat [3] proposed a pure integer programming to solve the CF problem considering machine reliability and alternative process routing. Their research shows that the reliability consideration has a significant impact on the overall system efficiency. Furthermore, the integration of CF problem with production planning and system reconfiguration has investigated by Kioon et al. [4]. A multi-objective mathematical model developed by Zhao and Wu [5]. The objectives of their proposed model are minimization of cell load variation, total inter–intra cell part trips and total number of exceptional elements. Because of complexity of given problem, they also implemented a genetic algorithm to solve the problem. Tavakkoli-Moghaddam et al. [6] developed a dynamic cell formation problem. In their work three basic meta-heuristics including genetic algorithm (GA), simulated annealing (SA) and Tabu search (TS) were implemented to solve the model and then these algorithms compared to each other. Also Wu et al. [7] proposed a water flow-like algorithm to solve the CF problem. Their meta-heuristic solution approach is verified in both solution effectiveness and efficiency aspects in comparison with other solution methods.

There are many researches in literature which are devoted to inter–intra cell layout problems. Most of these studies have considered the layout problem as a sequel to the CF problem because of the model complexity. Tavakkoli-Moghaddam et al. [8] proposed a new mathematical model to solve the inter–intra cell layout problem in presence of stochastic demands. In their research it is assumed that predefined CF structure is as an input of the inter–intra cell layout problem. Krishnan et al. [9] investigated three basic steps in the inter–intra cell layout problem while grouping the machines into cells is performed at first step in order to minimize total inter–intra cell part trips. The second step addresses two heuristic procedures for grouping the parts into the cells based on the machine grouping solution. At last a GA implemented to determine the best inter–intra cell layout.

Despite of previous mentioned studies, there are some researches which are proposed for simultaneous optimization of the CF and inter–intra cell layout problems. Kia et al. [10] proposed a SA algorithm to solve the CF and inter–intra cell layout problems simultaneously. Also Jolai et al. [11] proposed an electromagnetism-like meta-heuristic to solve the CF problem integrated with the inter–intra cell layout. Wu et al. [12] proposed a mathematical model which integrates CF, inter–intra cell layout and group scheduling problems. Because of its complexity, a GA with two heuristic operators is introduced. Arkat et al. [13] proposed two mathematical models to design a CMS. The first model was based on the integration of CF and group layout (GL) problems. The second model was integration of the first model which group scheduling (GS) problem which improved total system efficiency.

There are some studies which have considered operator related issues in a CM environment. Satuglu and Suresh [14] investigated the operator issues consideration in the hybrid CM environment. It involved three main steps. First, the parts with erratic demands are selected, as special parts to be processed in a functional layout of shop floor. Next, a mathematical model was proposed to solve the CF problem and at last step, operator assignment problem solved using a goal programming approach. Integration of CF problem with production planning and worker assignment in a dynamic environment is investigated by Mahdavi et al. [15]. Also Aryanezhad et al. [16] developed a new mathematical model which deals with concurrent solving of CF and operator assignment problems. Part routing flexibility, machine flexibility and also promotion of workers from one skill level to another were considered in their research.

Table 1 summarizes previous recent studies which have dealt with two or three aspects of CMS problem. According to the Table 1, it can be realized that there is not any attempt to solve the CF, GL and operator assignment problems concurrently. However, as pointed by Wu et al. [12] these problems are interrelated and their interactions should be considered in order to achieve an optimal solution.

This paper fills the gap by proposing a new integrated mathematical model. The aim of presented model is to minimize the total cost including inter–intra cell part trips, machine relocation, hiring, firing and salary of operators in a dynamic environment. Also since the proposed model includes non commensurable objectives, a LP-metric approach has been implemented to find the most preferred solution.

The rest of presented paper is organized as follows: In Section 2, a non-linear mathematical model based on mentioned objectives is proposed and two linearization techniques are implemented to simplify the model. Also the LP-metric approach

Table 1
The summary of literature review.

Study	Types of problem				Approaches	
	Cell formation	Group layout	Group scheduling	Operator assignment	Concurrent	Sequential
Tavakkoli -Moghaddam et al. [8]	*	*				*
Krishnan et al. [9]	*	*				*
Kia et al. [10]	*	*			*	
Jolai et al. [11]	*	*			*	
Wu et al. [12]	*	*	*		*	
Arkat et al. [13]	*	*	*		*	
Satuglu and Suresh [14]	*			*		*
Mahdavi et al. [15]	*			*	*	
Aryanezhad et al. [16]	*			*	*	
Presented paper	*	*		*	*	

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implemented in this paper will be discussed in this section. In Section 3, the proposed model efficiency will be verified by four hypothetical numerical examples and a real case study. Moreover; the sensitivity analysis will be conducted followed by conclusion in Section 4.

2. The mathematical model

2.1. Problem description and crucial assumptions

In this paper a non-linear mathematical model based on three main decisions of cell formation, operator assignment and inter-cell layout, is proposed. The major assumptions of the proposed model can be categorized into three groups based on three mentioned problems:

Cell formation assumptions

1. Demand of each part type in each production period, number of cells and lower–upper bounds of cell capacity, are predefined and constant over planning horizon.
2. Part trips and Machine relocation costs depend on the inter-cell layout.
3. Part and machine transmissions take zero time.
4. Machines operating and purchasing costs are not considered.
5. Each machine can perform only one operation in a moment.
6. The process sequence of each part type is determined and is constant in each production period (routing flexibility is not considered).

Operator assignment assumptions

7. An operator can be assigned to only one cell. The operator transmission between cells is not allowed.
8. An operator can be assigned to more than one machine based on his/her ability.
9. An operator can be trained to operate with specific machine in a production period by spending a training cost.
10. An operator can be hired or fired in each period independently.
11. Training is performed between periods and it takes zero time.

Inter-cell layout assumptions

12. Number of cell candidate locations is predetermined and it is constant over planning horizon.
13. Cell establishment cost and time are assumed to be zero.

2.2. Notation

Indices and their relative upper bounds

I	Number of machines
J	Number of parts
C	Number of machine cells should be constructed
G	Number of candidate locations to be a cell ($G \geqslant C$)
T	Number of production periods
OP_j^t	Number of operations required by part j in period t
K	number of available operators
i, i'	Index for machines ($i = 1, \ldots, I$)
j	Index for parts ($j = 1, \ldots, J$)
c, c'	Index for machine cells ($c = 1, \ldots, C$)
t	Index for production periods ($t = 1, \ldots, T$)
g, g'	Index for a candidate locations to be a cell ($g = 1, \ldots, G$)
d	Index for operations required by part j in period t ($d = 1, \ldots, OP_j^t$)
k	Index for operators ($k = 1, \ldots, K$)
Input parameters:	
a_{ki}	Training cost for operator k to operate with machine i
$Z_{ki}^1 = \begin{cases} 1; & \text{If worker } k \text{ is unable to operate with machine } i \text{ in first period} \\ 0; & \text{Otherwise} \end{cases}$	
D_j^t	Demand value for part j in period t
B_j^t	Handling batch size for part j in period t
$dis_{gg'}$	Distances between two candidate locations g and g'

(*continued on next page*)

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$P^t_{jdi} = \begin{cases} 1; & \text{If operation } d \text{ of part } j \text{ be processed by machine } i \text{ in time period } t \\ 0; & \text{Otherwise} \end{cases}$

W^t_{jd}	Processing time of operation d of part j in period t
Sa_{ki}	Salary for operator k to operate with machine i (per hour)
H_k	Hiring cost of operator k
F_k	Firing cost of operator k
$\min h$	Minimum number of operators should be hired in each production period
u_c, l_c	The upper and lower machine capacity for cell c
u_i, l_i	The maximum and minimum number of operators required by machine i
u_k, l_k	The maximum and minimum number of machines which can be assigned to operator k

Decision variables:

$$X^t_{ic} = \begin{cases} 1; & \text{If machine } i \text{ is assigned to cell } c \text{ in period } t, \\ 0; & \text{Otherwise,} \end{cases}$$

$$Y^t_{cg} = \begin{cases} 1; & \text{If cell } c \text{ is located in location } g \text{ in period } t, \\ 0; & \text{Otherwise,} \end{cases}$$

$$h^t_k = \begin{cases} 1; & \text{If operator } k \text{ is hired in period } t, \\ 0; & \text{Otherwise,} \end{cases}$$

$$r^t_{ki} = \begin{cases} 1; & \text{If operator } k \text{ is assigned to machine } i \text{ in period } t, \\ 0; & \text{Otherwise,} \end{cases}$$

$$S^t_{kc} = \begin{cases} 1; & \text{If operator } k \text{ is assigned to cell } c \text{ in period } t, \\ 0; & \text{Otherwise,} \end{cases}$$

$$Z^t_{ki} = \begin{cases} 1; & \text{If operator } k \text{ is unable to operate with machine } i \text{ in period } t \ (t \geqslant 2), \\ 0; & \text{Otherwise.} \end{cases}$$

2.3. Objective function and constraints

The 0–1 non linear programming model for the CMS design is presented as follows:

$$\min\ OB = \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{d=1}^{OP^t_j-1}\sum_{i=1}^{I}\sum_{i'=1}^{I}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{g=1}^{G}\sum_{g'=1}^{G}\left[\frac{D^t_j}{B_j}\right] dis_{gg'} P^t_{jdi} P^t_{j(d+1)i'} \max(X^t_{ic}Y^t_{cg} + X^t_{i'c'}Y^t_{c'g'} - 1, 0) \quad (1 - 1)$$

$$+ \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{d=1}^{OP^t_j-1}\sum_{c=1}^{C}\left[\frac{D^t_j}{B_j}\right] \max\left(\sum_{i=1}^{I} P^t_{jdi}X^t_{ic} + \sum_{i=1}^{I} P^t_{j(d+1)i}X^t_{ic} - 1, 0\right) \quad (1 - 2)$$

$$+ \sum_{t=1}^{T-1}\sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{c'=1}^{I}\sum_{g=1}^{G}\sum_{g'=1}^{G} \max(X^t_{ic}Y^t_{cg} + X^{t+1}_{ic'}Y^{t+1}_{c'g'} - 1, 0) dis_{gg'} \quad (1 - 3)$$

$$+ \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K} h^t_k r^t_{ki} X^t_{ic} S^t_{kc} Z^t_{ki} a_{ki} \quad (1 - 4)$$

$$+ \sum_{t=1}^{T}\sum_{k=1}^{K} (h^t_k H_k + (1 - h^t_k) F_k) \quad (1 - 5)$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{d=1}^{OP^t_j}\sum_{i=1}^{I}\sum_{k=1}^{K} P^t_{jdi} D^t_j W^t_{jd} r^t_{ki} Sa_{ki}. \quad (1\text{-}6)$$

Subjected to :

$$\sum_{c=1}^{C} X^t_{ic} = 1 \quad \forall i, t; \quad (2)$$

$$\sum_{i=1}^{I} X_{ic}^{t} \leqslant u_c \quad \forall c, t; \tag{3}$$

$$\sum_{i=1}^{I} X_{ic}^{t} \geqslant L_c \quad \forall c, t; \tag{4}$$

$$\sum_{k=1}^{K} h_{k}^{t} \geqslant \min h \quad \forall t; \tag{5}$$

$$r_{ki}^{t} \leqslant h_{k}^{t} \qquad \forall k, i, t; \tag{6}$$

$$S_{kc}^{t} \leqslant h_{k}^{t} \qquad \forall k, c, t; \tag{7}$$

$$\sum_{k=1}^{K} r_{ki}^{t} \leqslant u_i \quad \forall i, t; \tag{8}$$

$$\sum_{k=1}^{K} r_{ki}^{t} \geqslant l_i \quad \forall i, t; \tag{9}$$

$$\sum_{i=1}^{I} r_{ki}^{t} \leqslant h_{k}^{t} u_k \quad \forall k, t; \tag{10}$$

$$\sum_{i=1}^{I} r_{ki}^{t} \geqslant h_{k}^{t} l_k \quad \forall k, t; \tag{11}$$

$$r_{ki}^{t} \leqslant \sum_{c=1}^{C} X_{ic}^{t} S_{kc}^{t} \quad \forall k, i, t \tag{12}$$

$$\sum_{c=1}^{C} S_{kc}^{t} = h_{k}^{t} \quad \forall k, t; \tag{13}$$

$$Z_{ki}^{t+1} = (1 - r_{ki}^{t}) \times Z_{ki}^{t} \quad \forall t = 1, \ldots, T-1, \ \forall k, i; \tag{14}$$

$$\sum_{g=1}^{G} Y_{cg}^{t} = 1 \quad \forall t, c; \tag{15}$$

$$\sum_{c=1}^{C} Y_{cg}^{t} \leqslant 1 \quad \forall t, g; \tag{16}$$

$$r, S, h, Z, X, Y \in \{0, 1\}. \tag{17}$$

The objective function consists of two main cost categories. The first one is related to part movement costs and the second one is about operator related issues. The first term of objective function (1-1) minimizes the inter-cell part movement. Term (1-2) minimizes intra-cell part movements. The third term of the objective function minimizes system reconfiguration cost. Term (1-4) in the objective function minimizes operators training costs. Term (1-5) is related to operators' hiring and firing costs. Finally operator's total salary is minimized by the last term of the objective function.

Constraint (2) is to ensure that each machine should be assigned to only one cell. Cells capacity is constrained by constraints (3) and (4). Constraint (5) ensures that the minimum numbers of operators are hired. Constraints (6) and (7) state that an operator can be assigned to a machine and a cell, respectively, if has been hired in that period. Minimum and maximum number of operators required by each machine is restricted by constraints (8) and (9), respectively. The maximum and minimum number of machines that each operator can operate with is restricted by constraints (10) and (11), respectively. Constraint (12) ensures that an operator can be assigned to a machine in a same cell. Actually this constraint restricts the operator transmission between cells. Constraint (13) guarantees that each hired operator should be assigned to only one cell. Training effect is taken into account by constraint (14) and it states that the trained operator in a period will not need to learn again to work with the same machine. Each cell should be assigned to only one candidate location and a location

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can be opened only for one cell. These constraints are stated by constraints (15) and (16), respectively. At last constraint (17) defines variables type where all are binary variables.

2.4. Linearization

The mathematical model proposed in this paper is a non-linear model because of terms (1-1)–(1-4) and constraints (12) and (14). As lots of exact solution approaches have been developed for linear models, the proposed model should be reformulated as a pure 0–1 linear programming model by introducing some new variables with auxiliary constraints for solving the problem in a reasonable computational time. In order to reach a linear model by minimum number of required constraints, some techniques are implemented in two steps:

Step 1

Consider the pure quadratic 0–1 term $Z = X_1 \times X_2 \times \cdots \times X_n$ where X_i $(i = 1, \ldots, n)$ is a binary variable. It is obvious that Z can be 1 if and only if all the variables are 1 and otherwise it must be 0. Considering this mathematical point, following method can be applied by introducing some new auxiliary constraints:

$$Z \leqslant X_i \quad \forall i = 1, \ldots, n,$$
$$Z \geqslant \sum_{i=1}^{n} X_i - (n-1).$$

This type of nonlinearity is came into view in terms (1-1), (1-3), (1-4) and set constraints (12) and (14). So let define new binary variables XY_{icg}^t, RZ_{ki}^t, XS_{ikc}^t, Q_{ikc}^t which are computed by following equations:

$$XY_{icg}^t = X_{ic}^t Y_{cg}^t \quad \forall i, c, g, t;$$
$$RZ_{ki}^t = r_{ki}^t Z_{ki}^t \quad \forall i, k, t;$$
$$XS_{ikc}^t = X_{ic}^t S_{kc}^t \quad \forall i, k, c, t;$$
$$Q_{ikc}^t = h_k^t XS_{ikc}^t RZ_{ki}^t \quad \forall i, k, c, t.$$

By considering these equations, the following auxiliary constraints should be added to the proposed model:

$$XY_{icg}^t \leqslant X_{ic}^t \quad \forall i, c, g, t, \tag{18}$$

$$XY_{icg}^t \leqslant Y_{cg}^t \quad \forall i, c, g, t, \tag{19}$$

$$XY_{icg}^t \leqslant X_{ic}^t + Y_{cg}^t - 1 \quad \forall i, c, g, t, \tag{20}$$

$$RZ_{ki}^t \leqslant r_{ki}^t \quad \forall i, k, t, \tag{21}$$

$$RZ_{ki}^t \leqslant Z_{ki}^t \quad \forall i, k, t, \tag{22}$$

$$RZ_{ki}^t \geqslant r_{ki}^t + Z_{ki}^t - 1 \quad \forall i, k, t, \tag{23}$$

$$XS_{ikc}^t \leqslant X_{ic}^t \quad \forall i, k, c, t, \tag{24}$$

$$XS_{ikc}^t \leqslant S_{kc}^t \quad \forall i, k, c, t, \tag{25}$$

$$XS_{ikc}^t \geqslant X_{ic}^t + S_{kc}^t - 1 \quad \forall i, k, c, t, \tag{26}$$

$$Q_{ikc}^t \leqslant h_k^t \quad \forall i, k, c, t, \tag{27}$$

$$Q_{ikc}^t \leqslant XS_{ikc}^t \quad \forall i, k, c, t, \tag{28}$$

$$Q_{ikc}^t \leqslant RZ_{ki}^t \quad \forall i, k, c, t, \tag{29}$$

$$Q_{ikc}^t \geqslant h_k^t + XS_{ikc}^t + RZ_{ki}^t - 2 \quad \forall i, k, c, t. \tag{30}$$

Step 2

The max function in terms (1-1)–(1-3), can be linearized by replacing an additional variable and two auxiliary constraints as follows:

$$\begin{array}{lcl} \min\ T & & \min\ Z \\ St: & \rightarrow & St: \\ T = \max(X,0) & & Z \geqslant X \\ & & Z \geqslant 0 \end{array}$$

By using this technique, let define new binary variables $M^t_{icgi'c'g'}$, N^t_{jdc}, $E^t_{icc'gg'}$ which are replaced by following equations:

$$M^t_{icgi'c'g'} = \max(XY^t_{icg} + XY^t_{i'c'g'} - 1, 0) \quad \forall i, i', c, c', g, g', t,$$

$$N^t_{jdc} = \max\left(\sum_{i=1}^{I} P^t_{jdi} X^t_{ic} + \sum_{i=1}^{I} P^t_{j(d+1)i} X^t_{ic} - 1, 0\right) \quad \forall j, d, c, t,$$

$$E^t_{icc'gg'} = \max(XY^t_{icg} + XY^{t+1}_{ic'g'} - 1, 0) \quad \forall i, c, c', g, g', t.$$

By these considerations, also six auxiliary constraints should be added to the proposed model as follows:

$$M^t_{icgi'c'g'} \geqslant XY^t_{icg} + XY^t_{i'c'g'} - 1 \quad \forall i, i', c, c', g, g', t, \tag{31}$$

$$M^t_{icgi'c'g'} \geqslant 0 \quad \forall i, i', c, c', g, g', t, \tag{32}$$

$$N^t_{jdc} \geqslant \sum_{i=1}^{I} P^t_{jdi} X^t_{ic} + \sum_{i=1}^{I} P^t_{j(d+1)i} X^t_{ic} - 1 \quad \forall j, d, c, t, \tag{33}$$

$$N^t_{jdc} \geqslant 0 \quad \forall j, d, c, t, \tag{34}$$

$$E^t_{icc'gg'} \geqslant XY^t_{icg} + XY^{t+1}_{ic'g'} - 1 \quad \forall i, c, c', g, g', t, \tag{35}$$

$$E^t_{icc'gg'} \geqslant 0 \quad \forall i, c, c', g, g', t. \tag{36}$$

Thus, the final version of the linear 0–1 programming model can be presented as follows:

$$\min\ OB = \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{d=1}^{OP^t_j - 1}\sum_{i=1}^{I}\sum_{i'=1}^{I}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{g=1}^{G}\sum_{g'=1}^{G}\left\lceil\frac{D^t_j}{B_j}\right\rceil dis_{gg'} P^t_{jdi} P^t_{j(d+1)i'} M^t_{icgi'c'g'} \tag{1 - 7}$$

$$+ \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{d=1}^{OP^t_j - 1}\sum_{c=1}^{C}\left\lceil\frac{D^t_j}{B_j}\right\rceil N^t_{jdc} \tag{1 - 8}$$

$$+ \sum_{t=1}^{T-1}\sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{c'=1}^{I}\sum_{g=1}^{G}\sum_{g'=1}^{G} E^t_{icc'gg'} dis_{gg'} \tag{1 - 9}$$

$$+ \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K} Q^t_{ikc} a_{ki} \tag{1 - 10}$$

$$+ \sum_{t=1}^{T}\sum_{k=1}^{K} (h^t_k H_k + (1 - h^t_k) F_k) \tag{1 - 5}$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{d=1}^{OP^t_j}\sum_{i=1}^{I}\sum_{k=1}^{K} P^t_{jdi} D^t_j W^t_{jd} r^t_{ki} Sa_{ki}. \tag{1-6}$$

Subjected to:

Unaltered set constraints (2)–(11), (13), (15), and (16), new auxiliary constraints (18)–(36) and also:Set constraint (12) is replaced by:

$$r^t_{ki} \leqslant \sum_{c=1}^{C} XS^t_{ikc} \quad \forall k, i, t. \tag{37}$$

Set constraint (14) is replaced by:

$$Z^{t+1}_{ki} = Z^t_{ki} - RZ^t_{ki} \quad \forall t = 1, \ldots, T - 1,\ \forall k, i. \tag{38}$$

Set constraint (17) is replaced by:

$$r, X, S, h, Y, Z, XY, XS, RZ, Q, M, N, E \in \{0, 1\} \tag{39}$$

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Table 2
The number of binary variables in the linear model.

Variable	Count	Variable	Count	Variable	Cont
r_{ki}^{t}	$K \times I \times T$	Y_{cg}^{t}	$C \times G \times T$	$M_{icgi'c'g'}^{t}$	$I^2 \times C^2 \times G^2 \times T$
S_{kc}^{t}	$K \times C \times T$	XY_{icg}^{t}	$I \times C \times G \times T$	N_{jdc}^{t}	$J \times OP_j^t \times C \times T$
h_{k}^{t}	$K \times T$	RZ_{ki}^{t}	$K \times I \times T$	$E_{icc'gg'}^{t}$	$I \times C^2 \times G^2 \times T$
Z_{ki}^{t}	$K \times I \times T$	XS_{ikc}^{t}	$I \times K \times C \times T$		
X_{ic}^{t}	$I \times C \times T$	Q_{ikc}^{t}	$I \times K \times C \times T$		

Sum $= 3(K \times I \times T) + (K \times C \times T) + (K \times T) + (I \times C \times T) + (C \times G \times T) + (I \times C \times G \times T)$
$+2(I \times K \times C \times T) + (I^2 \times C^2 \times G^2 \times T) + (J \times OP_j^t \times C \times T) + (I \times C^2 \times G^2 \times T)$

Table 3
The number of constraints in the linear model.

Constraint	Count	Constraint	Count	Constraint	Count
(2)	$I \times T$	(16)	$G \times T$	(29)	$I \times C \times K \times T$
(3)	$C \times T$	(18)	$I \times C \times G \times T$	(30)	$I \times C \times K \times T$
(4)	$C \times T$	(19)	$I \times C \times G \times T$	(31)	$I^2 \times C^2 \times G^2 \times T$
(5)	T	(20)	$I \times C \times G \times T$	(32)	$I^2 \times C^2 \times G^2 \times T$
(6)	$K \times I \times T$	(21)	$I \times K \times T$	(33)	$J \times OP_j^t \times C \times T$
(7)	$K \times C \times T$	(22)	$I \times K \times T$	(34)	$J \times OP_j^t \times C \times T$
(8)	$I \times T$	(23)	$I \times K \times T$	(35)	$I \times C^2 \times G^2 \times T$
(9)	$I \times T$	(24)	$I \times C \times K \times T$	(36)	$I \times C^2 \times G^2 \times T$
(10)	$K \times T$	(25)	$I \times C \times K \times T$	(37)	$K \times I \times T$
(11)	$K \times T$	(26)	$I \times C \times K \times T$	(38)	$K \times I \times (T-1)$
(13)	$K \times T$	(27)	$I \times C \times K \times T$		
(15)	$C \times T$	(28)	$I \times C \times K \times T$		

Sum $= 3(I \times T) + 3(C \times T) + T + 5(K \times I \times T) + (K \times C \times T) + 3(K \times T) + (G \times T) + 3(I \times C \times G \times T) + 7(I \times C \times K \times T)$
$+2(I^2 \times C^2 \times G^2 \times T) + 2(J \times OP_j^t \times C \times T) + 2(I \times C^2 \times G^2 \times T) + (K \times I \times (T-1)) +$ number of constraints relate to constraint (39)

Total number of variables and constraints in the proposed linear 0–1 programming model are reported in Tables 2 and 3, respectively. In Table 3, the constraints related to constraint (39) are not counted.

2.5. LP-metric approach

Generally, LP-metric method provides a broader principle of compromise for solving multiple criteria decision making problems. It transfers m-objectives (criteria), which are conflicting and non commensurable into one objective through normalizing the objectives and Pareto optimal solutions can be obtained using the single aggregated objective function. Consider the vector of objective functions as $F(x) = (f_1(x), f_2(x), \ldots, f_n(x))$ and the ideal vector of these functions as $F*(x) = (f_1^*(x), f_2^*(x), \ldots, f_n^*(x))$ and also the anti-ideal vector of objective functions as $F^-(x) = (f_1^-(x), f_2^-(x), \ldots, f_n^-(x))$, where $f_i^*(x)$ and $f_i^-(x)$ are positive and negative ideal solutions for ith objective function, respectively. LP-metric defines the distance between two points $F(x)$ and $F*(x)$ according to Eq. (40). Actually in order to commensurate the units of objective functions this metric can be used:

$$D = \left(\sum_{i=1}^{n} \lambda_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right)^p \right)^{\frac{1}{p}}, \quad p = 1, 2, \ldots, \tag{40}$$

where λ_i is the importance weight of the objective function i. The overall goal is to minimize distance function (D) according to problem constraints and find the Pareto optimal solutions.

2.6. Necessity of simultaneous consideration of different decisions in a CMS

The objective function of the proposed mathematical model consists of two basic costs. The first one is related to the machine and part related costs including inter-cell part trips (term (1-7)), intra-cell part trips (term (1-8)) and machine relocation cost (term (1-9)). The second one is related to operator related issues and includes training cost (term (1-10)), hiring and firing cost (term (1-5)) and salary cost (term (1-6)). In order to analyze the model more precisely, two basic costs are named as f_1 and f_2, respectively. Based on this definition let consider three separate models as follows:

Model 1:
Minimize $OB1$ = Summation of terms (1-7)–(1-9).

Subjected to: constraints (2)–(4), (15), (16), (18)–(20), (31)–(36), (39).

Model 2:
Minimize $OB2$ = Summation of terms (1-10), (1-5), (1-6).
Subjected to: constraints (5)–(11), (13), (21)–(30), (37)–(39).

Model 3:
Minimize OB = Summation of $OB1$ and $OB2$ using LP-metric distance.
Subjected to: (2)–(11), (13), (15), (16), (18)–(39).

By this decomposition models 1 and 2 can be treated as separate optimization models which try to reduce the cell formation, inter cell layout and operator related costs, respectively. But by implementing model 3 which is formulated in this paper, three main decisions of cellular manufacturing system including CF, GL and operator assignment can be optimized simultaneously.

In order to verify performance of the proposed model, let define the following notations:

f_1^* The optimal objective value of model 1.
f_2^* The optimal objective value of model 2.
f_3^* The optimal objective value of model 3.
f_{13}^* The objective value of model 1 which is obtained by replacement of optimal variables of model 3 into model 1.

Aryanezhad et al. [16] suggested a criterion as Eq. (41) to calculate the gap of differences between the optimal objective value of model 1 and its objective value by considering optimal solutions of model 3.

$$I = \frac{|f_1^* - f_{13}^*|}{f_{13}^*} \times 100. \quad (41)$$

Also they have shown that $f_3^* \leqslant f_1^* + f_2^*$. Hence two different conditions can be occurred. If $f_3^* = f_1^* + f_2^*$ solving the models 1 and 2, consecutively, can be a good strategy instead of solving of model 3 with more complexity. However it has been shown that in most of cases $f_3^* < f_1^* + f_2^*$. In this situation solving the model 3 in order to find an optimal solution which satisfies both models 1 and 2 can be selected as a decision strategy. In this study the mentioned criterion in Eq. (41) is used to show the ratio of total objective improvement by incorporating both decisions simultaneously.

Despite of considering two main separate objectives in model 3, it is worth to pay attention that their scales are different and cannot be integrated originally, so the LP-metric technique which normalizes the objective functions is proposed. Then the model 3 can be reformulated as model 4 (we consider P = 1).*Model 4:*Minimize $LP = \lambda_1\left(\frac{f_1^* - f_1(x)}{f_1^* - f_1^-}\right) + \lambda_2\left(\frac{f_2^* - f_2(x)}{f_2^* - f_2^-}\right)$.Subjected to: (2)–(11), (13), (15), (16), (18)–(39).

3. Numerical illustration

In order to verify and validate the proposed model, some numerical examples are generated by hypothetical parameters given in Table 4 and are solved by branch-and-bound (B&B) approach using Lingo 8.0 software which have run on a PC including Core i5 and 1 GB RAM. According to the model 4 and considering of $(\lambda_1, \lambda_2) = (0.5, 0.5)$, number of inter–intra cell part trips, number of machine relocations and also operator related costs are reported in Table 5. However, the first and last instances which are small size and large size examples, respectively can be analyzed more precisely.

3.1. Instance 1

This example includes two cells, four machines, five available operators, two production periods and four parts should be processed by machines considering their operation sequence in each period. Table 6 reports the machine-part related infor-

Table 4
Different instances with hypothetical parameters.

Example number	Parts	Machines	Cells	Periods	Operators	Salary(Integer Uniform)	Hiring and firing(Integer Uniform)	Operation time (s)(Uniform)	Training cost(Integer Uniform)
Instance 1	4	4	2	2	5	(1,9)	(10,100)	(0.1,0.7)	(1,10)
Instance 2	5	4	2	1	10	(1,9)	(10,100)	(0.1,0.7)	(1,10)
Instance 3	8	6	3	1	10	(1,9)	(10,100)	(0.1,0.7)	(1,10)
Instance 4	8	7	3	1	10	(1,9)	(10,100)	(0.1,0.7)	(1,10)

Table 5
Objective values obtained by solving model 4.

Example number	Number of inter–intra cell Part trips	Operator related costs
Instance 1	294	768
Instance 2	144	769
Instance 3	447	802
Instance 4	561	981

Table 6
The input information of part-machine matrix of instance 1.

Parts	Period 1				Period 2			
	Process sequence	Processing time of each operation (W)	Demand (D_j^1)	Batch size	Process sequence	Processing time of each operation (W)	Demand (D_j^2)	Batch size
1	1–4–3	0.2, 0.1, 0.05	100	10	4–2	0.2, 0.07	200	10
2	1–3	0.2, 0.05	60	10	4–2–3–1	0.2, 0.1, 0.08, 0.05	160	10
3	4–2–1	0.15, 0.1, 0.05	80	10	4–2–3	0.15, 0.1, 0.05	180	10
4	2–1–3	0.1, 0.08, 0.05	90	10	3–1–2–4	0.15, 0.1, 0.08, 0.05	20	10

Table 7
Capabilities of operators in working with different machines $(1 - Z)$ – instance 1.

Operators	Machine			
	1	2	3	4
1	0	0	1	1
2	0	1	0	0
3	0	0	1	0
4	0	1	0	0
5	1	0	1	0

Table 8
Training cost of operators to learn working with different machines (a) – instance 1.

Operators	Machine			
	1	2	3	4
1	7	6	0	0
2	5	0	4	6
3	3	3	0	5
4	5	0	3	4
5	0	5	0	5

mation of the first instance. The operator-machine related input data including capabilities of operators in operating with different machines, training, salary, hiring and firing costs for instance 1 are reported in Tables 7-10. For example, as shown in Table 7, the operator 5 is able to work with machines 1 and 3. The minimum and maximum operators required by each machine are 2 ($L_i = 2, U_i = 2$). Table 11 shows the distance between candidate cell locations. Moreover, it is assumed that the minimum and maximum machine capacities of each cell are 1 and 2, respectively ($L_c = 1, U_c = 2$).

Pay-off matrix which shows the positive and negative ideal solutions for each objective function of model 4, are reported in Table 12.

According to the Tables 2 and 3, the model 4 for the instance 1 includes 1877 binary variables and 3794 linear constraints which could find the optimal solution in 6 s. The solution of this example including operator assignment solution and also cell formation and inter-cell layout solutions are reported in Tables 13 and 14.

Hence the objective functions and problem assumptions are different from previous investigations; it cannot be compared with results of previous studies. However; the obtained solution can be interpreted as follows: As shown in Table 14, operator 3 is assigned to machine 2 and since this operator cannot work with this machine, he should be trained and the training cost for this operator-machine pair is 3. It is realized from Table 7 that operator 4 can be assigned to this machine without any training cost. But the salary value decreases 7 units for an hour (Table 9). Since operator 4 who is trained only for machine 2, is not employed in the first production period.

Table 9
Salary of operators in working with different machines (*Sa*) – instance 1.

Operators	Machine			
	1	2	3	4
1	12	11	9	10
2	9	12	10	11
3	10	9	13	14
4	11	16	9	12
5	12	11	11	11

Table 10
Hiring (*H*) and firing (*F*) costs, maximum and minimum number of machines to be assigned for each operator – instance 1.

Operators	Hiring cost	Firing cost	U_k	L_k
1	20	15	2	1
2	17	15	2	1
3	20	10	2	1
4	18	15	2	1
5	15	12	2	1

Table 11
Distance between cell locations (*Dis*) – instance 1.

From	To		
	1	2	3
1	0	5	7
2	5	0	2
3	7	2	0

Table 12
Pay off matrix of two objectives – instance 1.

	Positive Ideal solution value		Negative Ideal Solution value	
	f_1	f_2	f_1	f_2
f_1	$f_1^* = 144$	20872	$f_1^- = 20872$	20872
f_2	844.43	$f_2^* = 768.04$	870.85	$f_2^- = 932.27$

Table 13
Inter-cell layout and machine grouping – instance 1.

Cells	Period 1		Period 2	
	Cell location	Machines	Cell location	Machines
1	3	1, 2	2	2, 4
2	2	4, 3	3	1, 3

Table 14
Operator assignment based on cell formation solution – instance 1.

Operators	Machines (period 1)				Machines (period 2)			
	1	2	3	4	1	2	3	4
1			*	*				*
2	*	*			*		*	
3	*	*				*		
4					*		*	
5			*	*		*		*

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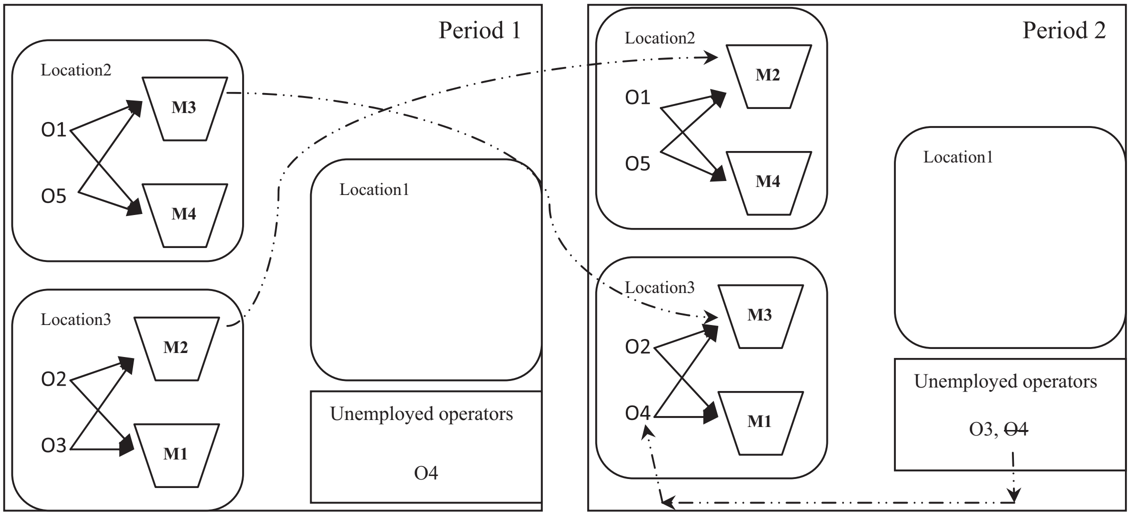

Fig. 1. The schematic view of cell formation, inter-cell layout and operator assignment – instance 1.

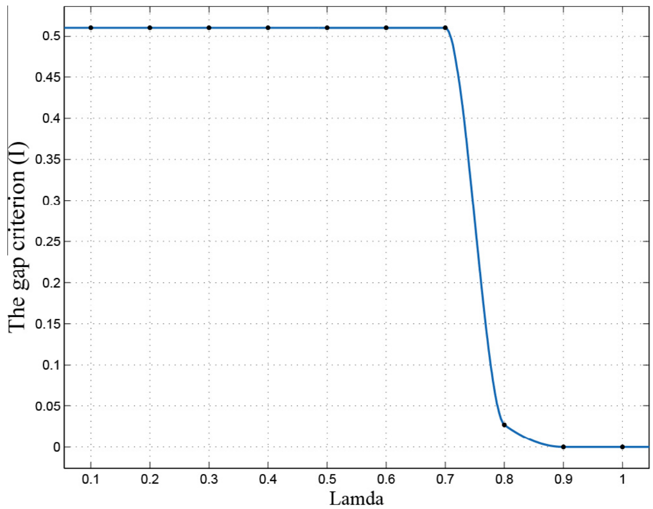

Fig. 2. The effect of different λ_1 values on the gap criterion (I) – instance 1.

Also the schematic view of production plan which is obtained for this example is depicted in Fig. 1. It is clear that erratic demands and different processing sequence in periods 1 and 2, results in different inter-cell layout, cell formation and operator assignment solutions in each period. The calculated I criterion for this example is *0.51* which means that when the operator assignment problem is taken into account, the total cost is decreased 51%.

Also the sensitivity analysis of incorporating the operator related issues into the cellular manufacturing system is obtained using different values of λ_1. Fig. 2 demonstrates the results. Different values of λ_1 and its effect on the I criterion value has been depicted in this figure. According to this figure for larger values of λ_1 ($\lambda_1 \geqslant 0.9$) the cell formation of both models 1 and 4 behave the same. But by increasing of λ_2 values the operator assignment solution will be significant and by $\lambda_1 \leqslant 0.7$ the maximum differences between models 1 and 4 is obtained in the first instance.

3.2. Instance 4

This example includes seven machines, three cells, ten available operators, two production periods and eight parts which should be processed by different machines considering the manufacturing process sequence. Machine-part related information for instance 4 is illustrated in Tables 15 and 16. Furthermore; operator related issues including operators' capabilities in doing different jobs, training, salary, hiring and firing costs are reported in Tables 17–20. Table 21 shows the distance between candidate cell locations. Moreover, it is assumed that the minimum and maximum machine capacities of each cell are 1 and 3, respectively ($L_c = 1$, $U_c = 3$).

Table 15
The input information of part-machine matrix – instance 4 (period 1).

Parts	Process sequence	Processing time of each operation (W)	Demand (D_j^1)	Batch size
1	1–3–4–2–7–6	0.6, 0.3, 0.5, 0.7, 0.5, 0.4	90	10
2	5–6	0.1, 0.3	100	10
3	6–5–2–7–4–3	0.6, 0.1, 0.2, 0.2, 0.6, 0.6	20	10
4	2–4–5	0.7, 0.3, 0.6	100	10
5	1–2–7–5–3	0.5, 0.7, 0.5, 0.5, 0.5	70	10
6	1–2–5	0.1, 0.4, 0.3	10	10
7	7–6–1–4–7–5–3–2	0.1, 0.4, 0.2, 0.5, 0.1 0.7, 0.6, 0.6	30	10
8	5–2–7–6–4–3–7	0.1, 0.1, 0.4, 0.5, 0.1, 0.7,0.4	60	10

Table 16
The input information of part-machine matrix – instance 4 (period 2).

Parts	Process sequence	Processing time of each operation (W)	Demand (D_j^1)	Batch size
1	2–5	0.7, 0.03	20	10
2	7–6	0.5, 0.4	50	10
3	6–7	0.1, 0.2	100	10
4	5–7–3–7	0.1, 0.5, 0.3, 0.5	80	10
5	6–1–7	0.2, 0.6, 0.5	100	10
6	5–3–7–1	0.6, 0.2, 0.6, 0.2	70	10
7	6–3–5–2–7–1	0.3, 0.6, 0.7, 0.6, 0.4, 0.4	10	10
8	5–1–2–7–3–7	0.1, 0.5, 0.2, 0.1, 0.4, 0.1	90	10

Table 17
Capabilities of operators in working with different machines – instance 4 ($1 - Z$).

Operators	Machine						
	1	2	3	4	5	6	7
1	1	0	0	0	0	1	0
2	0	1	0	1	1	0	0
3	1	1	0	0	0	0	0
4	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0
6	0	1	0	0	0	0	1
7	1	1	0	0	0	0	0
8	0	0	0	0	0	1	0
9	1	1	0	0	0	0	0
10	0	1	0	0	0	0	0

Table 18
Training cost of operators to learn working with different machines – instance 4 (a).

Operators	Machine						
	1	2	3	4	5	6	7
1	0	7	9	5	6	0	6
2	8	0	9	0	0	4	5
3	0	0	9	4	6	4	6
4	7	6	8	3	6	3	4
5	0	7	8	5	6	4	6
6	6	0	9	4	6	3	0
7	0	0	7	5	6	4	5
8	6	5	9	5	6	0	4
9	0	0	9	4	6	4	6
10	6	0	7	3	6	3	4

Table 22 illustrates the optimal layout of cells in periods 1 and 2 and also machine grouping within cells. Since a large value of machine relocation cost has been considered for this instance, it is clear that machines are not moved between cells.

Tables 23 and 24 show the operator assignment to machines in both periods. In order to obtain this optimal solution, operators 1, 2, 4, 5, 6, 8 should be trained to work with machines 4, (3,7), 6, 6, (1,4) and (3,7), respectively. Considering these

Table 19
Salary of operators in working with different machines – instance 4 (*Sa*).

Operators	Machine						
	1	2	3	4	5	6	7
1	9	5	8	1	7	10	7
2	10	9	4	5	2	6	5
3	2	2	7	4	2	2	4
4	10	5	2	8	5	2	9
5	7	10	8	8	10	3	6
6	1	8	1	2	4	9	6
7	3	10	3	5	6	3	10
8	6	7	1	5	3	9	3
9	10	1	1	7	8	3	8
10	10	9	9	8	3	10	8

Table 20
Hiring (*H*) and firing (*F*) costs, maximum and minimum number of machines to be assigned for each operator – instance 4.

Operators	Hiring cost	Firing cost	U_k	L_k
1	100	80	3	1
2	100	80	3	1
3	80	60	3	1
4	40	20	3	1
5	30	10	3	1
6	40	20	3	1
7	50	20	3	1
8	50	20	3	1
9	50	20	3	1
10	50	25	3	1

Table 21
Distance between cell locations (*Dis*) – instance 4.

From	To		
	1	2	3
1	0	5	7
2	5	0	2
3	7	2	0

Table 22
Inter-cell layout and machine grouping obtained – instance 4.

Cells	Period 1		Period 2	
	Cell location	Machines	Cell location	Machines
1	3	1, 2, 4	1	6
2	2	3, 5, 7	3	1, 2, 4
3	1	6	2	3, 5, 7

tables it can be realized that operators 7 and 10 in period1 and operators 1, 7 and 10 in period 2 are not employed in order to the optimal system efficiency be obtained.

As this example is a large-sized problem, the optimal solution has been obtained after 1416 min. however, the computational time of the first example is 6 s.

As pointed by Dimopoulos and Zalzala [17] the CMS design problem is NP-hard which means increasing in problem size will increase the computational time by the non-polynomial function. So, Linearization of mathematical models is an important issue especially in large sized problems. In order to evaluate the performance of linear mathematical model versus the non-linear model in both optimality and computational time, the instances given in Table 4 are solved by both linear and nonlinear models. The schematic comparison of computational time of examples solved by both linear and nonlinear models

Table 23
Operator assignment in period 1 based on cell formation solution – instance 4.

Operators	Machines						
	1	2	3	4	5	6	7
1				*			
2			*		*		*
3	*	*					
4						*	
5						*	
6	*			*			
7							
8			*		*		*
9		*					
10							

Table 24
Operator assignment in period 2 based on cell formation solution – instance 4.

Operators	Machines						
	1	2	3	4	5	6	7
1							
2			*		*		*
3	*	*		*			
4						*	
5						*	
6	*			*			
7							
8			*		*		*
9		*					
10							

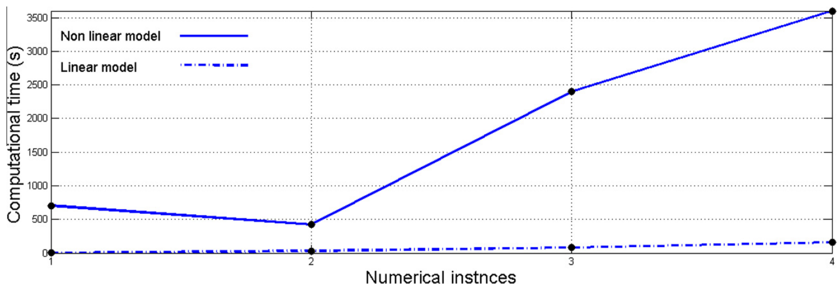

Fig. 3. The comparison of linear and non-linear mathematical models in computational time aspect instance 1–4.

is demonstrated in Fig. 3. According to this figure the computational time of linear model has lower slope by increasing the problem size in comparison to the nonlinear form.

3.3. An application on real data

A CMS can be designed in manufacturing companies with small and medium size machines. These kinds of machines can be relocated considering many real world manufacturing costs. Iranian automobile industry is composed of domestic cars produced by two major companies, Saipa and Iran-Khodro in addition to a number of limited imported cars. In order to illustrate the applicability of proposed model, it was applied on data of design & manufacturing (die shop) department of SAIPA Co. This factory works 8 h/day, 30 days/month, and 12 months a year, which results in 2880 h/year as available capacity. In this department different kinds of parts such as pin, pierce punch, bottom die, guide and pallet guide pin should be manufactured based on processing sequence on different machines such as drilling machine, CNC milling machine, conventional milling machine, electro-erosion machine, etc. Considering a production horizon including two production periods (two months), the machine-part related information is reported in Table 25. Since the exact information of operator related issues were not in access, the estimated information is reported in Tables 26 and 27. Moreover; the salary of operators, firing and

Table 25
The process sequence (process time) of parts, demand, number of operators – case study.

	M1 (cutting)	M2 (conventional lathe)	M3 (milling)	M4 (grinding)	M5 (heating)	M6 (CNC milling)	M7 (tool grinding)	Demand period 1	Demand period 2
1. Plate guide pin	1(1)	2(7)			3(2)			400	300
2. Pierce punch	1(0.5)	3(7)	7(7)	5(20)	4(3)	2(3)	6(3)	200	400
3. Block D01		1–3(0.5-0.5)	2–4(5–7)		5(3)			200	100
4. Stop D02		1–3(0.5–1)	2–4(3–7)					400	200
5. Guide	1(1)	2–6(40–10)		4(28)	3(5)		5(3)	3000	2000
6. Stop D01		1–3(0.5–1)	2–4(3–7)					400	600
7. Shaft D03	1(0.5)	2(12)	3(3)		4(2)			400	700
8. Bottom Die		1–3(0.5–3)			4(2)	2(20)	5(3)	100	200
9. Pierce punch	1(0.5)	2–7(3–4)		5(15)	4(3)	3(15)	6(2)	200	0
10. Pin D08	1(0.5)	2(2)		4(1)	3(.5)			200	400
11. Pin P01	1(0.5)	2(5)	3(8)		4(1)			400	600
12. Master gage	1(1)	2(7)	3–5(4–5)		4(2)			1200	600

Table 26
Capabilities of operators in working with different machines $(1 - Z)$ – case study.

Operators	Machine						
	1	2	3	4	5	6	7
1		1	1				
2				1			1
3	1						
4					1		
5						1	1
6	1	1	1	1	1		
7	1	1	1				
8				1	1	1	1
9	1	1		1	1		
10						1	
11			1				1
12	1						
13		1	1	1	1		
14					1	1	1

Table 27
Training cost of operators to learn working with different machines (a) – case study.

Operators	Machine						
	1	2	3	4	5	6	7
1	500,00	0	0	80,000	12,000	–	–
2	80,000	120,000	110,000	0	–	–	0
3	0	20,000	30,000	15,000	10,000	15,000	20,000
4	30,000	20,000	5000	10,000	0	60,000	20,000
5	50,000	50,000	80,000	90,000	–	0	0
6	0	0	0	0	0	–	–
7	0	0	0	50,000	60,000	35,000	20,000
8	60,000	50,000	75,000	0	0	0	0
9	0	0	90,000	0	0	40,000	15,000
10	–	–	–	–	–	0	20,000
11	–	–	0	50,000	–	120,000	0
12	0	–	–	–	–	–	–
13	30,000	0	0	0	0	110,000	20,000
14	–	–	–	40,000	–	–	–
Number of operators required by each machine	2	2	1	2	3	2	1

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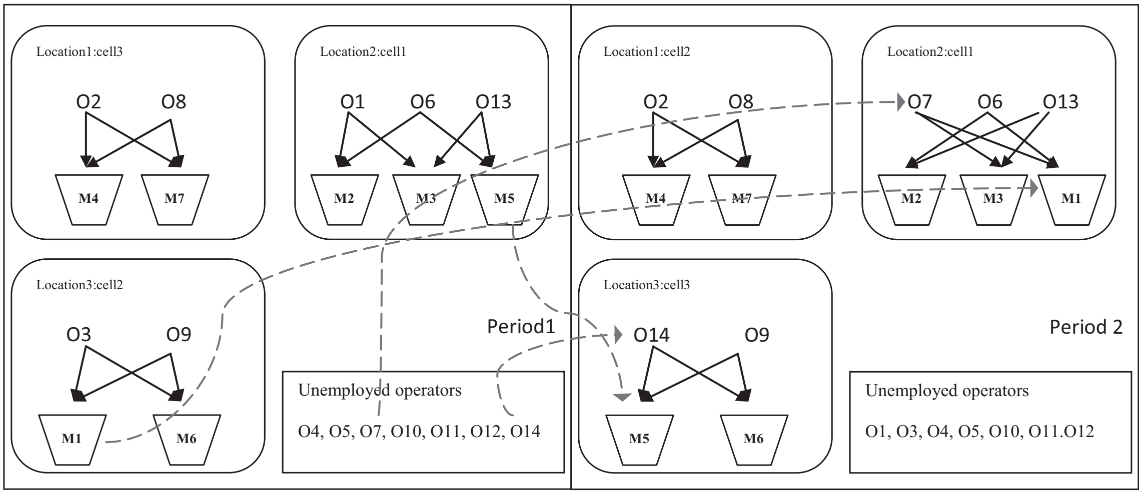

Fig. 4. The schematic view of cell formation, inter-cell layout and operator assignment case study.

hiring costs are supposed to be 900,000, 1,500,000, and 60,000, respectively. Also in this department each operator should be assigned to 2 machines in a production period. Also the machine relocation cost is assumed to be 50,000.

Fig. 4 illustrates the schematic view of optimal solution obtained for this problem. According to this figure it can be realized that operators 10, 11 and 12 are not hired in both periods. This is because of their incapability in working with different machines. Moreover; since the machine relocation cost has a large value, only machines 1 and 5 are moved between cells. This solution can be applied in mentioned department as an optimal manufacturing system. The optimal solution is found after 1653 min.

4. Conclusion

In this paper simultaneous consideration of the cell formation problem with inter-cell layout and operator assignment problems in a dynamic cellular manufacturing environment is investigated. A new mathematical model is proposed based on the consideration of these three sub problems, concurrently. To validate and verify the proposed mathematical model, some numerical examples are generated with hypothetical parameters. The results show that the proposed integrated model is more efficient than separate previous models. Also the sensitivity analysis of the coefficients confirms that the proposed model can find optimum points of separate objective functions. Also the applicability of proposed model is verifies by applying it on Saipa company as a car producer. However, since the proposed model is NP-hard such a way that solving a large size problem in a reasonable computational time is intractable, a Meta-heuristic solution approach can be beneficial in improving its time requirements. Also consideration other production elements such as machine reliability and duplication and also incorporating the intra-cell layout problem in provided framework can be interesting as future studies.

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