A Survey and Classification on Application of Coding Techniques in PAPR Reduction

Taha ValizadehAslani, Abolfazl Falahati

***Abstract*—Orthogonal frequency-division multiplexing (OFDM) is employed in various communication systems because of its high bandwidth efficiency and good performance in wireless communication channels; however, the major shortcoming of this system is the high peak-to-average power ratio (PAPR) of the output signal. This high value of the signal can exceed the linear range of power amplifier and lead to nonlinear distortion. Various methods are proposed in the literature to address this problem. Among which, channel coding techniques enjoy the inherent benefit of error correction. In this survey, primarily a brief overview of OFDM system and the problem of PAPR are presented, and then different applications of channel coding approaches in PAPR reduction are reviewed and compared. Moreover, in order to provide a better insight, a classification of such methods is presented. It will be shown that no coding-based PAPR reduction technique can achieve an acceptable error correction performance, a good PAPR reduction, and a low computational burden simultaneously; particularly, when the number of subcarriers increases.**

***Index Terms*— Orthogonal frequency-division multiplexing, peak-to-average power ratio, channel coding**

I. Introduction

IN the modern era, the demand for high-speed communication is increasing immensely. On the other hand, the market needs are trending towards wireless systems. Orthogonal frequency-division multiplexing (OFDM) not only achieves a great spectral efficiency but also performs very well in terms of robustness against intersymbol interference and frequency selectivity in a wireless channel. In addition, the complexity of the channel equalization is much lower in OFDM when compared with single carrier systems [١]–[٣]. For such profound advantages, OFDM is employed in different ubiquitous communication protocols such as IEEE ٨٠٢,١١ (Wireless LAN) [٤], IEEE ٨٠٢,١٦ (Wireless Man) [٥], Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), Digital Subscriber Line (DSL) and Long Term Evolution (LTE) systems [٣].

Analogous to many other electronic innovations, OFDM was first introduced in the mid-١٩٠٠s [٦], [٧] and continued to develop afterward [٢], [٨]–[١٠]. In OFDM, different orthogonal subcarriers modulate a stream of data using a signal constellation such as phase shift keying (PSK) or quadrature amplitude modulation (QAM). Unlike the conventional frequency division multiplexing (FDM), the subcarriers overlap in the frequency domain (Fig. ١). Indeed, due to the orthogonality of subcarriers, their independence is guaranteed. This characteristic saves a portion of bandwidth in OFDM. Moreover, since each subcarrier experiences only a narrow frequency band, a frequency-selective fading channel acts as a channel with flat frequency response. So, signaling in a wireless channel becomes possible without a complex equalization technique.

In spite of all the aforementioned advantages, OFDM suffers from a major problem of high peaks in the envelope of the output signal. These peaks are resulting from the constructive superposition of different subcarriers and lead to a high peak-to-average power ratio (PAPR). If the output signal of the power amplifier (PA) does not follow the mentioned peaks due to saturation, the signal fidelity is reduced and error rate increases; moreover, the produced distortion in the output signal may produce out of band frequency components that can cause interference with the adjacent channels. On the other hand, in order to amplify the high peaks, an expensive PA with a large input back off is required, which is not economically viable and can also affect the battery usage negatively in portable devices. Furthermore, the problem of high PAPR deteriorates as the number of subcarriers increases, because of the increased chance of constructive superposition. Hence, a PAPR reduction technique is necessary for an efficient OFDM system with a large number of subcarriers.

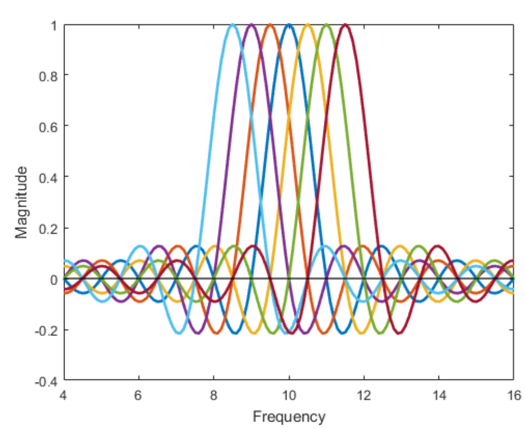

Fig. ١. Subcarriers' overlapping in frequency domain

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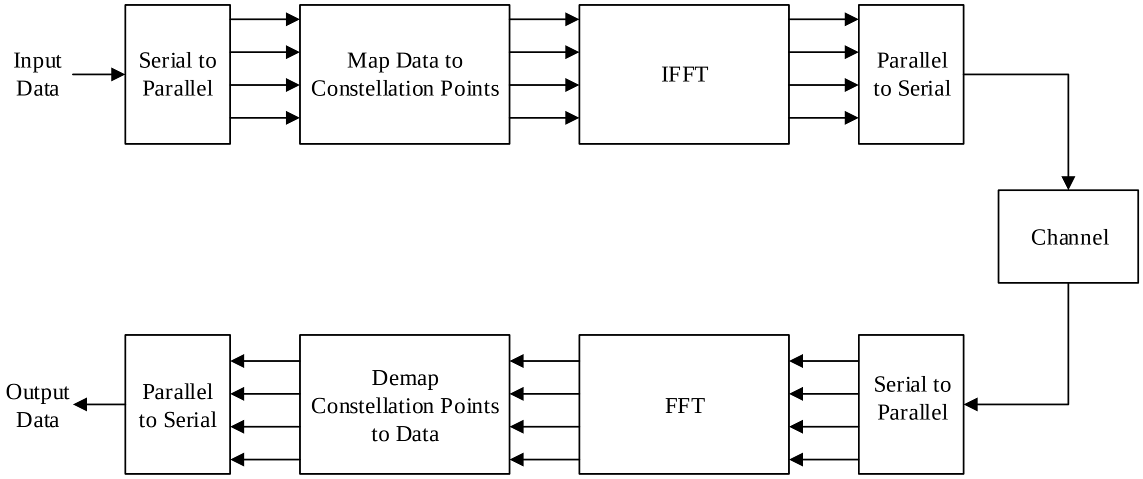

Fig. ٢. OFDM Transmitter and Receiver

Many methods are proposed for PAPR control [٣], [١١], [١٢]. Among them, coding-based techniques have the advantage of error control, which makes them an interesting choice for PAPR reduction. The rest of this survey is organized as follows. In section II, OFDM system model and the problem of PAPR are briefly discussed. Section III presents the most important part of this survey, which is a review of coding-based PAPR reduction techniques. The analysis and comparison of these methods are provided in section IV. Finally, the conclusion is presented in Section V.

II. System Model And PAPR

OFDM divides the available channel bandwidth into a number of overlapping subchannels, each of which is occupied by a subcarrier. To ensure their independence, the subcarriers are chosen to be mutually orthogonal. The overall OFDM symbol is defined as a sum of N such orthogonal subcarriers, each conveying a part of data in the form of a complex signal constellation point, in time interval of T. Here, signal constellation points are denoted as a_i, for $0 < i < N - ١$. When subcarriers are selected as complex exponentials, OFDM bandpass signal can be written as

$$x_{bp}(t) = \sum_{i=٠}^{N-١} a_i e^{j٢\pi(f_c + i\Delta f)t} \qquad (١)$$

where f_c is the frequency of the first carrier, and Δf is the frequency difference between each two adjacent subcarriers, and is chosen to be $١/T$ so that all subcarriers are orthogonal. The baseband signal can be obtained by a frequency shift

$$x(t) = e^{-j٢\pi f_c t} x_{bp}(t)$$

$$= e^{-j٢\pi f_c t} \sum_{i=٠}^{N-١} a_i e^{j٢\pi(f_c + i\Delta f)t}$$

$$= \sum_{i=٠}^{N-١} a_i e^{j٢\pi i \Delta f t} \qquad (٢)$$

Sampling $x(t)$ at rate of N samples per T produces the discrete baseband signal

$$x[n] = \sum_{i=٠}^{N-١} a_i e^{j٢\pi i n/N} \qquad (٣)$$

which is the exact formula of IDFT. Hence, OFDM signal can be generated by IDFT [٨]. At the receiver, the demodulation process has the same relation except for the negative sign of the exponentials, which can be represented via DFT. Since DFT and IDFT chipsets are readily available, especially in the form of fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT), the IDFT/DFT representation creates an effective and economical method to implement OFDM.

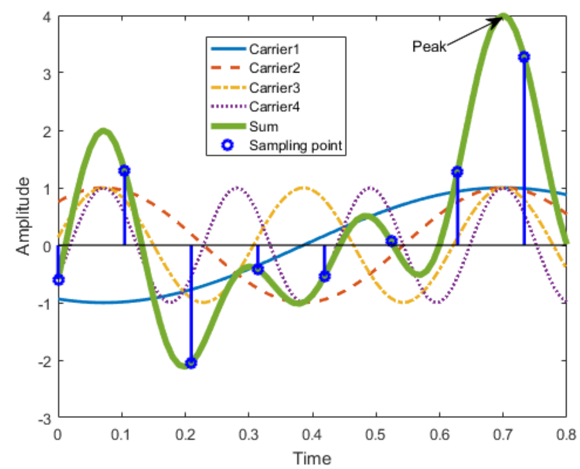

Fig. ٣. Constructive superposition of carriers and sampling

When the number of subcarriers increases, the chance of constructive superposition also increases, which leads to high amplitude peaks in the output signal. PAPR for each OFDM symbol is defined as the ratio of its maximum power to the average power.

$$PAPR\{x(t)\} = \max_{t\in(٠,T)} \frac{|x(t)|^٢}{E[|x(t)|^٢]} \qquad (٤)$$

here $E[.]$ represents expectation operator.

Some references [١٣], [١٤] use crest factor as a measure of peak to average ratio of the output signal, which is defined as

$$CF\{x(t)\} = \max_{t\in(٠,T)} \frac{|x(t)|}{x_{rms}} \qquad (٥)$$

where x_{rms} is the root mean square value of $x(t)$, and is the square root of $E[|x(t)|^٢]$. Thus, by definition

$$PAPR\{x(t)\} = CF\{x(t)\}^٢ \qquad (٦)$$

In this survey, PAPR is used as the measure of peak to average ratio of the OFDM signal.

If the amplitude of all signal constellation points is unity, the average power of OFDM signal can be calculated as

$$E[|x(t)|^{٢}] = E[x(t)x^{*}(t)] = E\left[\sum_{i=٠}^{N-١} a_i e^{j٢\pi i\Delta ft} \sum_{l=٠}^{N-١} {a_l}^{*} e^{-j٢\pi l\Delta ft}\right] = E\left[a_i {a_l}^{*} \sum_{i=٠}^{N-١}\sum_{l=٠}^{N-١} e^{j٢\pi i\Delta ft} e^{-j٢\pi l\Delta ft}\right] = N \quad (٧)$$

where x^{*} denotes complex conjugate of x. So, the average power is equal to the number of subcarriers. In the same manner, maximum possible power can be calculated

$$\max_{t\in(٠,t)} |x(t)|^{٢} = \max_{t\in(٠,t)} x(t)x^{*}(t) = \max_{t\in(٠,t)} \sum_{i=٠}^{N-١} a_i e^{j٢\pi i\Delta ft} \sum_{l=٠}^{N-١} {a_l}^{*} e^{-j٢\pi l\Delta ft} = \max_{t\in(٠,t)} \sum_{i=٠}^{N-١}\sum_{l=٠}^{N-١} a_i {a_l}^{*} e^{j٢\pi i\Delta ft} e^{-j٢\pi l\Delta ft} = N^{٢} \quad (٨)$$

Thus, if the amplitude of all signal constellation points is one, PAPR of OFDM signal is, at most, $N^{٢}/N$ or N; however, In a practical OFDM system, PAPR is usually lower than N [١٥].

Furthermore, when the output of IDFT represents the OFDM symbol, PAPR can be defined as

$$PAPR\{x[n]\} = \max_{n\in ١,٢,\dots,N-١} \frac{|x[n]|^{٢}}{E[|x[n]|^{٢}]} \quad (٩)$$

But this representation of PAPR can be spurious because, in the calculation of IDFT, the samples are acquired at the Nyquist rate, *i.e.* N samples per each OFDM symbol; however, the high peaks of the signal can be located in time instants other than sampling points. In this case, high peaks exist in real OFDM signal, but (٩) does not acknowledge their existence as shown in Fig. ٣. Thus, the OFDM signal must be oversampled by factor of $\alpha > ١$, so that the chance of undetected high peaks decreases [١٦]. Values of ٢،٨ and ٤ are suggested for oversampling factor α in [١٧] and [١٨] respectively.

For a large enough number of subcarriers, according to central limit theorem (CLT), van Nee [١٧] suggests that the Real and imaginary parts of the OFDM signal can be modeled as two Gaussian processes. This means, the signal envelope has a Rayleigh distribution, and its power follows a central chi-square distribution with ٢ degrees of freedom. Hence, the probability that PAPR of each sample is below a threshold δ, or equivalently, power cumulative distribution function (CDF) for each sample is

$$CDF(\delta) = ١ - e^{-\delta} \quad (١٠)$$

In (٩), PAPR for each OFDM symbol is defined as a maximum of PAPR for all samples, so in order to guarantee that PAPR of the OFDM symbol is below δ, PAPR of all samples must be below δ. If the signal samples are obtained at Nyquist rate, they are statistically independent and the probability of the event that PAPR of $x[n]$ does not exceed δ can be expressed as

$$Prob(PAPR\{x[n]\} < \delta) = (١ - e^{-\delta})^{N} \quad (١١)$$

Also, probability of event that PAPR exceeding threshold δ can be expressed as complementary cumulative distribution function (CCDF) by

$$Prob(PAPR\{x[n]\} > \delta) = ١ - (١ - e^{-\delta})^{N} \quad (١٢)$$

However, (١٢) does not necessarily hold for the PAPR of continuous OFDM signal defined in (٤), due to the possibility of occurrence of peaks in time instants other than sampling points. Reference [١٧] suggested an empirical modification of (١٢), in which it assumed that the samples remain independent after oversampling, so when oversampling factor is ٢،٨, (١٢) can be rewritten as

$$Prob(PAPR\{x[n]\} > \delta) = ١ - (١ - e^{-\delta})^{٢.٨N} \quad (١٣)$$

But the validity of (١٣) is rejected by [١٥] since it relies on an assumption for the support of which no evidence is provided; moreover, numerical results of [١٥] does not match with (١٣). Hence, another expression for CCDF of PAPR is derived analytically [١٥], for high thresholds, CCDF can be approximated as

$$Prob(PAPR\{x[n]\} > \delta) \cong e^{-N\sqrt{\frac{\pi}{٣}\delta}e^{-\delta}} \quad (١٤)$$

III. PAPR Reduction Techniques

Generally, PAPR reduction methods are categorized into two main classes: signal distortion and candidate generation. Signal distortion methods treat PAPR by distorting the signal, which results in increasing system bit error rate. On the other hand, candidate generation methods generate different candidates for each input data and transmit the one with lowest PAPR. Signal distortion methods include clipping the high peaks and then filtering the signal to avoid the resulting out of band distortion [١٩]–[٢٩] and companding the OFDM signal [٣٠]–[٤٤]. The main advantage of these methods is the implementation simplicity.

In candidate generation methods, such as selective mapping (SLM), PAPR is reduced by generating different candidates for each OFDM signal and transmitting the one with lowest PAPR. Different candidates can be generated via multiplying data by different phase shifts and choose the set of phase factors that produces the lowest PAPR [٤٥]–[٥٤]. Fig. ٤ illustrates the block diagram of SLM system. Similarly, in partial transmitted sequence (PTS) input data is partitioned into a number of non-overlapping sub-blocks, and IDFT is calculated for each sub-block. Then, each one is multiplied by a phase rotating factor. Phase factors are selected to reduce PAPR of the overall signal, which is formed by the concatenation of all sub-blocks [٥٥]–[٦٣]. In order to recover the original data, in both methods, the receiver must know which candidate is selected at the transmitter, so the number of the selected candidate must also be transmitted as extrinsic information; obviously, this occupies a part of channel capacity. Furthermore, an error in transmission of extrinsic information can lead to erroneous demodulation of the whole frame.

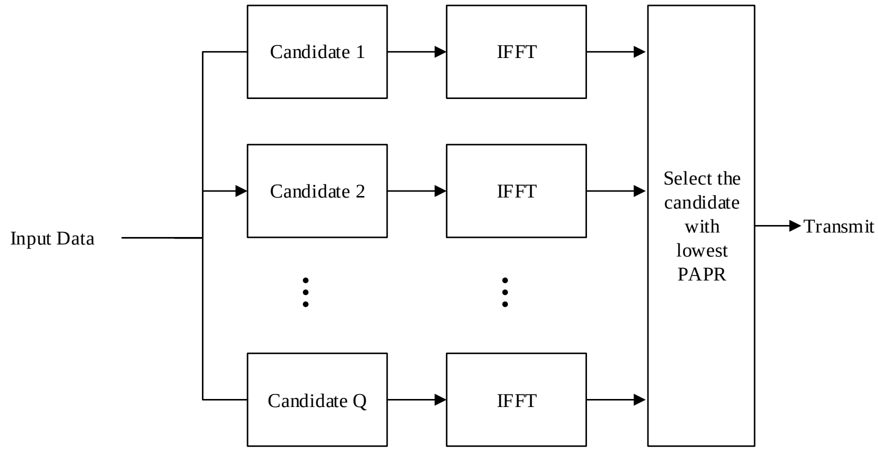

Fig. ٤. Candidate generation method for PAPR reduction

Tone reservation (TR) technique [٦٤]–[٧٣] allocates a portion of subcarriers to PAPR reduction instead of conveying information, and assigns values to those subcarrier amplitudes that minimize the PAPR of the OFDM signal. In tone injection (TI) method [٦٤], [٧٤]–[٧٦], the signal constellation size is increased, therefore the position of each point in the original constellation is mapped into several new equivalent points in the extended constellation, new positions can be chosen to reduce PAPR. Similarly, in active constellation extension (ACE) method [٧٧]–[٨١], some of the outer points in the signal constellation are actively extended towards outside the constellation when PAPR is high.

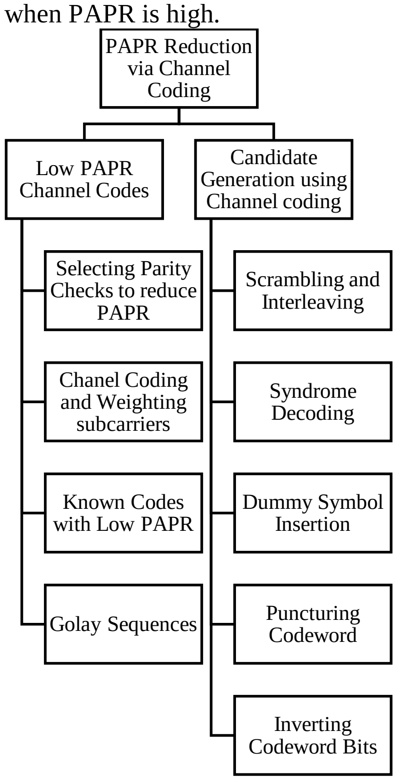

Fig. ٥. Classification of coding based PAPR reduction techniques

PAPR reduction via channel coding techniques is also proposed, which enjoys the inherent advantage of error control. Application of channel coding techniques in PAPR reduction can be classified into two categories: codes with low PAPR and candidate generation using channel coding techniques. Each is further classified into separate groups as shown in Fig. ٥. Explanation and comparison of these techniques is the main objective of this survey.

A. Low PAPR Channel Codes

Channel coding is an inseparable part of virtually all communication systems. It would be very interesting if this part also controls the PAPR so that no other unit is required for this purpose. The literature in this category is further classified into four groups. In some codes, one or several parity check bits are placed next to the codeword bits for PAPR control purpose. In other proposed methods, channel coding in conjunction with weighting subcarriers restricts PAPR. Some approaches, use known codes with good PAPR control performance. Last but not least, Golay complementary sequences can serve as an important class of PAPR control channel codes.

١) Selecting Parity Checks to Reduce PAPR

First attempt to control PAPR using a block code was contributed by Jones *et al* [٨٢] in ١٩٩٤. By computer simulation, it was observed that in a Binary Phase Shift Keying (BPSK) OFDM with only ٤ subcarriers, certain input patterns like ١١١١ and ١٠١٠ lead to a high PAPR. To avoid such patterns, the authors suggested a simple odd parity check, which created a (٣, ٤) code. By this rudimentary method, they successfully reduced PAPR by ٣,٥٤dB. In [٨٣], this method was further investigated with codes of different rates. The main drawback of this approach is the restriction of its application to OFDM systems with a small number of subcarriers and short code lengths. It must be noted that, when the length of codeword increases, high PAPR patterns cannot be avoided by a simple odd parity check, also choosing good codewords becomes a harder task for codes with larger lengths due to exponential growth in the number of possible choices.

Later on, in ١٩٩٦, Wulich [٨٤] generalized concept of [٨٢] by proposing a simple cyclic codeword with rate ¾ in an OFDM system where the number of subcarriers is a multiple of ٤. In this scheme, high PAPR patterns are avoided by mathematical analysis of OFDM waveform, and ٣dB reduction in PAPR can be achieved compared to the uncoded case, even when the number of subcarriers is increased. But this reduction is also insignificant for OFDM systems with a large number of subcarriers.

Fragiacomo *et al* [٨٥] observed that regardless of code length, high PAPR is generated when the number of odd and even bits is the same (*e.g.* ٠٠٠٠٠٠ and ٠١٠١٠١). So, they suggest a parity check bit at the end of the codewords, whose value is the inverse of the penultimate bit. Codewords with equal odd and even bits are avoided without sacrificing channel capacity, but, as it was shown in [٨٦], the resulting increase is negligible when frame size is large.

In order to control PAPR when the OFDM frame is large, [٨٧] proposes the sub-block-coding scheme, which in nutshell, is dividing long information sequence into several sub-blocks and adding one parity check bit to each. The authors also design another mechanism based on changing the position of the parity check bits. This issue will be discussed in later stages. Although it is not impossible to control PAPR of OFDM systems with large frames using this technique, the problem of code rate loss becomes severe when the frame size grows, since a very low code rate is required in order to limit PAPR.

Jiang and Zhu in [٨٨], [٨٩] suggest complement block coding: inserting one or more complement bits, which is the reverse of some of the information bits, in the middle of the information bits to create codewords. Their numerical analysis shows that complement block coding outperforms approaches of [٨٢], [٨٥], [٨٧] in terms of code rate and PAPR control.

٢) Channel Coding and Weighting Subcarriers

Within the concept of PAPR reduction, the method of exploiting channel coding and applying some modifications to subcarriers jointly is also suggested by many studies. These modifications include phase shifts and amplitude altering. Jones and Wilkinson [٩٠] combined weighted OFDM: subcarriers are weighted by predefined values, with an (٨،٤) linear block code and achieved a ٤dB gain in PAPR control, as well as error control. Tarokh and Jafarkhani [٩١] continued that approach by using phase shifting functions as weighting values. They optimized these shifts and achieved ٤,٥dB PAPR reduction. In [٩٢], a specific ٤×٨ generator matrix, as well as a specific phase shifter is designed to serve as a precoder that can limit the PAPR in an OFDM system with QPSK modulation and ٤ subcarriers. The generator matrix also adds ٤ parity check bits that can be used for error detection. Reference [٩٣] utilizes product codes to compensate for BER degradation resulting from the use of weighting functions to control PAPR. Such weighting functions were initially introduced in [٩٤] where different functions such as Gaussian or Chebyshev were employed to modify the amplitudes of the subcarriers in order to control PAPR. [٩٣] Employs codes that are in fact a product of ٢ BCH codes introduced in [٩٥]. Numerical analysis of [٩٣] reveals that a product BCH(١٥،١١) code that is exploited in conjunction with Gaussian weighting function technique achieves the best PAPR control performance.

٣) Known Channel codes with Inherent Low PAPR

There are also some channel codes that enjoy a good PAPR control performance inherently. Dual of Bose–Chaudhuri–Hocquenghem (dual BCH) codes have a nature of good PAPR control performance [٩٦], [٩٧], but were not employed in OFDM systems due to their poor error correcting performance. Sabbaghian *et al* in [٩٨], show that only dual BCH codes with the capability of correcting one error have a considerably good PAPR control performance, and propose a new code based on such codes. They utilize this new code jointly with a low-density parity-check (LDPC) code in a time-frequency turbo structure [٩٥] where, to ensure low PAPR codewords, the new code based on dual BCH code is exploited in frequency domain, and an LDPC code is used in time domain so that a good error control performance is achieved (Fig. ٦). Furthermore, the associated *maximum a posteriori* (MAP) decoder is also developed at the receiver end. The same method is also proposed for Reed-Muller and LDPC codes in [٩٩] which will be covered later.

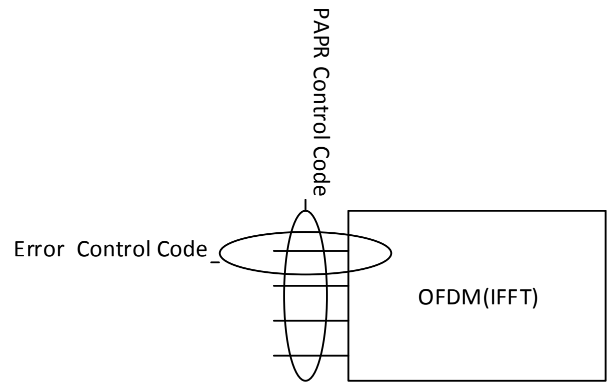

Fig. ٦. Separation of error control and PAPR control codes

Recently, other known codes such as Goppa codes [١٠٠], Hamming codes and cyclic codes [١٠١] are employed for PAPR control in OFDM system. But in their simulations, [١٠٠] and [١٠١] deploy very short codes and a small number of subcarriers. It must be noted that good performance of such codes there, cannot guarantee the same results when system dimensions increase.

٤) Golay complementary sequences and Reed-Muller codes

Golay complementary sequences were first proposed by Golay in [١٠٢], [١٠٣] for their application in Infra-Red spectrometry, and later were introduced to communication engineering society in [١٠٤]. For a bipolar sequence $\boldsymbol{a} = [a_٠, a_١, \ldots, a_{n-١}]$ of length N, where $a_i \in \{-١, ١\}$, autocorrelation function is defined as

$$R_a(k) = \sum_{i=٠}^{n-k-١} a_i a_{i+k} \qquad (١٥)$$

Two sequences of length n, say $\boldsymbol{a}$ and $\boldsymbol{b}$, are called Golay complementary sequences, if the following condition is satisfied [١٠٤]

$$R_a(k) + R_b(k) = \begin{cases} ٢N, & k = ٠ \\ ٠, & k \neq ٠ \end{cases} \qquad (١٦)$$

Such pair of sequences were calculated by Golay in [١٠٤] and independently by Shapiro in [١٠٥]. Construction of such sequences is investigated in different articles [١٠٦]–[١١٢].

Boyd in [١٣] employed Golay sequences to generate initial phases in a multitone signal in order to control PAPR. Later, in [١٣], [١١٣] an interesting property of Golay sequences was observed: For a Golay sequence $\boldsymbol{a}$, Fourier transform can be defined as

$$S_a(f) = \sum_{i=0}^{n-1} a_i e^{-j2\pi f i T_s} \qquad (١٧)$$

where T_s is the sampling period of the sequence $\boldsymbol{a}$. Note that, (١٧) is essentially the conjugate of OFDM baseband signal. By calculating the Fourier transform of (١٦), the following property of Power spectral density (PSD) of the sequences can be obtained

$$|S_a(f)|^{٢} + |S_b(f)|^{٢} = ٢N \qquad (١٨)$$

where the fact that PSD and autocorrelation function are Fourier transform couples is exploited. It immediately follows from (١٨) that

$$|S_a(f)| \leq \sqrt{٢N} \qquad (١٩)$$

By replacing f with t, an upper bound on OFDM baseband signal is achieved since this relation also holds for ${S_a}^*(f)$, and consequently for ${S_a}^*(t)$, which is by definition the OFDM baseband signal. Now, modified version of (١٩) and (٧) can be replaced in PAPR relation (٤). So

$$PAPR\big(S(t)\big) = max \frac{|S(t)|^{٢}}{E[|S(t)|^{٢}]} \leq \frac{٢N}{N} = ٢ \qquad (٢٠)$$

Thus, PAPR is bounded by the value of ٢ or ٣dB. Numerical analysis shows that this bound is often reached in a practical system. Hence, (١٩) and (٢٠) can be considered to be tight bounds [١٤], [١١٣].

The aforementioned analysis proves a restriction on PAPR value when a Golay sequence is employed as the input data in OFDM signal; however, this method cannot be employed in a practical OFDM system since the modulating sequence bears no information. In order to transmit information with such sequences, van Nee [١١٤] in ١٩٩٦, used different complementary sequences with a large minimum Euclidean distance as different codewords. In his method, PAPR is controlled strictly, since each codeword is a complementary sequence. At the same time, the minimum distance between different codewords is utilized for error correction at the receiver. However, the process of search for good codewords becomes arduous when code length increases.

A major theoretical breakthrough regarding the application of Golay sequences in coding theory was achieved when Davis and Jedwab reported the relationship between such sequences and second order Reed-Muller codes in [١١٥], [١١٦] and presented it fully in [١٠٦], [١١٧]. Reed-Muller codes are a class of linear error correction codes that were discovered by Muller [١١٨], but their decoder was designed by Reed [١١٩]. Davis and Jedwab showed that Golay sequences can be obtained from Reed-Muller codewords. Hence, an OFDM signal can be modulated with Golay sequences when it bears information. Suppose that $\boldsymbol{X}_{٠}, \boldsymbol{X}_{١} \dots \boldsymbol{X}_n$ are binary sequences of length ٢^m, which are rows of a generator matrix for the first order Reed-Muller code $RM(١, m)$. These sequences with their element by element products $\boldsymbol{X}_i\boldsymbol{X}_j$ $(١ \leq i < j \leq m)$ together constitute rows of second-order Reed-Muller code $RM(٢, m)$. What Davis and Jedwab proved is the code

$$\sum_{i=١}^{m-١} \boldsymbol{X}_{\pi(l)}\boldsymbol{X}_{\pi(l+١)} + \sum_{i=٠}^{m} c_i\boldsymbol{X}_i \qquad (٢١)$$

whose codewords are binary Golay sequences for $c_i \in \{٠,١\}$ and any permutation π of $\{١,٢ \dots m\}$. As $m!/٢$ distinct permutations exist, $\lfloor log_٢(\frac{m!}{٢})\rfloor$ (where $\lfloor . \rfloor$ indicates floor function) bits can be coded via first sum, and another $m + ١$ bits can be directly used as c_is in the second sum. Therefore, a total of "$\left\lfloor log_٢\left(\frac{m!}{٢}\right)\right\rfloor + m + ١$" bits can be coded in a second-order Reed-Muller code, where coded bits form a Golay sequence. A similar method is also presented with non-binary polyphase Golay sequences, when Phase Shift Keying (PSK) signal constellations is employed [١٠٦]. Paterson in [١٢٠], [١٢١], develops a unified mathematical theory that coveres the results of Davis and Jedwab as a special case.

For the sake of achieving a greater transmission rate, Rößing and Tarokh [١٢٢] constructed ١٦-Quadrature Amplitude Modulation (١٦-QAM) Golay complementary sequences by viewing ١٦-QAM as the addition of two scaled ٤–PSK signal constellations. This scenario enhances the data rate at a cost of increasing the upper bound of PAPR to ٣,٦ (٥,٥٦ dB). Later on, Chong, Venkataramani and Tarokh in [١٢٣] constructed ١٦-QAM Golay sequences with PAPR values of ٢,٦ (٤,١dB) and ٢ (٣dB). Authors of [١٢٤] utilize Golay sequences for general square M-QAM constellations with low PAPR, where M is a power of ٤, and derive different bounds on PAPR for different choices of M. Their approach is applied to STAR-١٦-QAM constellation where PAPR is proven to be bounded by ٢ (٣dB) [١٢٥], [١٢٦]. The Golay sequences are employed to control PAPR of Asterisk-١٦-QAM family, where the same PAPR bound is achieved [١٢٧], [١٢٨]. The method of viewing ١٦-QAM as ٢ QPSKs, as performed in [١٢٣], is extended to viewing ٦٤-QAM as a weighted sum of ٣ QPSKs in [١٢٩], [١٣٠]. There, PAPR is bounded by ٤,٦٦ (٦.٦dB). [١٠٨] suggests another method for construction of ٦٤-QAM Golay sequences. Li [١٠٧], a general ٤^q-QAM Golay sequences are constructed based on the observation made in [١٢٣], [١٢٩] that a ٤^q-QAM can be considered as a weighted sum of q QPSKs. PAPR for a general ٤^q-QAM Golay sequence is proved to be bounded by $٦(٢^q - ١)/(٢^q + ١)$, which approaches the value of ٦ (٧,٧dB) as constellation size grows [١٠٧].

Although Reed-Muller codes have error correcting capability, their error correcting performance is far from Shannon limit. This motivated Sabbaghian *et al* [٩٩] to use such RM codes together with a good error correcting code in a turbo block code structure [٩٥], [١٣١], [١٣٢]. In [٩٩], RM codes are employed in the frequency domain, *i.e.* on subcarriers, to control PAPR. And an LDPC (or BCH) code is employed in the time domain to guarantee a good error control performance (Fig. ٦).

The main advantage of Golay complementary sequences is that the PAPR is analytically smaller than a certain bound, unlike other methods which reduce the chance of PAPR exceeding a threshold, for any number of subcarriers.

However, when code length (and subsequently the number of subcarriers) increases, the problem of code rate loss becomes severe. Consider the binary Golay sequence of (٢١), the length of the RM codeword is ٢^{m}, and number of the information bits is "$\left\lfloor log_{٢}\left(\frac{m!}{٢}\right)\right\rfloor + m + ١$". Thus, the code rate is

$$R(m) = \frac{\left\lfloor log_{٢}\left(\frac{m!}{٢}\right)\right\rfloor + m + ١}{٢^{m}} \qquad (٢٢)$$

Since in (٢٢) the denominator grows faster than nominator, code rate approaches zero as code length increases. Indeed, this phenomenon can be observed in other different Golay sequences too [١٢١], [١٢٣], [١٣٠]. In order to address this problem, [١٣٣] proposes a new family of QAM Golay complementary sequences, which deal with the code rate loss problem partially, but even when these new sequences are used, an acceptable code rate cannot be achieved for OFDM systems with a large number of subcarriers.

Another approach to mitigate the problem of code rate loss is employing near complementary sequences (NCS) instead of Golay sequences. Such sequences were originally proposed by Parker and Tellambura in ٢٠٠١ [١٣٤], [١٣٥]. NCS are two sequences that form a near complementary pair (NCP), *i.e.* their autocorrelation function is not strictly zero for all non-zero shifts, and sum up to small values at some nonzero shifts. By relaxing the condition of zero autocorrelation, a greater set of sequences can be exploited for PAPR reduction. Even though the resulting PAPR is not bounded by ٢ anymore, the achieved freedom of choice can be exploited for increasing the code rate.

Authors of [١٣٦] propose a technique for construction of cosets of a *q*-ary generalization of the first-order Reed-Muller code. Such cosets contain the near complementary sequences and their PAPR is between ٢ and ٤. Reference [١٣٧] propose new ١٦-QAM near complementary sequences of length ٢^{m}, with PAPR of at most ٢,٤, that are built using non-linear offsets. In [١٣٨], a new family of *q*-ary near complementary sequences of various lengths and PAPR of at most ٤ is presented. Two types of ٦٤-QAM near complementary sequences, with PAPR < ٣,٦٢ and PAPR < ٢,٤٨, are also presented in [١٣٩].

Almost complementary sequences (ACS) are also proposed for mitigation of the code rate loss problem. Two sequences are called almost complementary if their correlation is zero except for one positive and one negative shift (τ and –τ). Such sequences were first introduced in [١٤٠], where authors constructed sequences of length ٢^{m} with PAPR of at most ٣. Reference [١٤١] proposes a new method for construction of ١٦-QAM almost complementary sequences, in which the PAPR is no greater than ٢,٤.

In addition to the code rate decrease, the problem of search for the good codes becomes a tremendous computational task when code length increases. For such reasons, the usefulness of Golay complementary sequences in PAPR control is limited to OFDM systems with a small number of subcarriers.

٥) Bounds on PAPR of Coded OFDM

Information theory can be exploited as a mean to investigate the possibilities and limitations of good codes that could lead to a good error correcting capability and a good PAPR control performance simultaneously. The results of such studies can put the design of new codes in perspective, and provide a mathematical framework for the future studies.

The first asymptotic limit on PAPR of coded OFDM was proposed in [١٤٢]. Based on extrapolation of numerical results and a theoretical speculation, the authors suggested that by choosing codewords with low peak power, PAPR can asymptotically be limited between ٢ and ٣ for a large number of subcarriers. They also claimed that by only employing ٤ redundancy bits, the PAPR would be necessarily within ١٠٪ of this limit. However, no analytical proof for this claim was presented. This claim was later rejected in [٩٦], [١٤٣] and [١٤٤].

The relation between PAPR and minimum Euclidean distance of a constant energy code (*i.e.* codes in which all symbols have equal energy such as PSK modulation scheme) was first investigated in ٢٠٠٠ in [٩١]. In the same year, Paterson and Tarokh [٩٦], [١٤٥] showed that for a constant energy code, there is a fundamental tradeoff between PAPR, code rate R and minimum Euclidean distance d_{min}. Thus, codewords with low PAPR have necessarily low minimum distance. In other words, the redundancy that is introduced by choosing only those codewords that have low PAPR cannot be employed optimally for the goal of error correction. Therefore, reduction in PAPR thorough channel coding leads to a decrease in minimum distance and/or code rate. This analytically derived theorem disproves the claim of [١٤٢] that PAPR of arbitrarily large code can be limited using only ٤ redundancy bits.

The authors of [٩٦] also show that, for any $R > ٠$ and $\Delta > ٠$, if R and Δ satisfy the following relation

$$٢^{R}\sqrt{٢\Delta(١-\frac{\Delta}{٢})} < ١ \qquad (٢٣)$$

then for all large enough n, an asymptotically good constant energy code of length n with rate of R and minimum Euclidean distance of $d_{min} = \sqrt{٢\Delta n}$ exists with PAPR of at most $٨logn$.

Other lower bounds on PAPR of long codes is proved in [١٤٣], [١٤٦], and later in [١٤٤]. All the authors agree that PAPR of good codes of length n is of order of $logn$. Hence, as code length grows, PAPR grows logarithmically unless code rate is decreased drastically.

B. Candidate Generation Using Channel Coding Techniques

As it mentioned earlier, one approach to limit the PAPR is to generate different OFDM signal candidates, all of which represent the same information, and transmit the one with the least PAPR. Channel coding techniques can be utilized to generate different candidates, and, in some cases, both error correction and PAPR control can be achieved. Also, in some channel coding PAPR reduction techniques, the redundancy of the code can be exploited to recover the original data without extrinsic information about the selected candidate. Different methods are proposed for candidate generation by channel coding. In what follows, these methods will be discussed and compared.

١) Scrambling and Interleaving

In ١٩٩٦, Van Eetvelt *et al* [١٤٧] used ٤ different scramblers to generate different OFDM symbols, all representing the same information, then transmit the one with lowest PAPR. In this method, the data indicating the selected scramblers is also transmitted alongside the information bits, so that the receiver can unscramble the signal. Similarly, in [١٤٨], [١٤٩], different interleavers are employed for candidate generation. In [١٥٠], [١٥١], different cyclic information shifts are utilized in conjunction with PTS in order to generate further candidates. In [١٥٢], different scramblers are used prior to a convolutional coder so that, they both can be integrated into a single device controlling both error and PAPR.

A novel method is introduced in [١٥٣]. The authors suggest a turbo code with Q distinct interleavers to create Q different candidates. The one with the lowest PAPR is transmitted without any extrinsic information. The receiver has Q different turbo decoders corresponding to Q different interleavers of the transmitter. To find the selected interleaver, the i-th decoder computes the reliability function for the k-th information bit in r-th iteration as

$$\tilde{L}_i^r(d_k) = log\left[\frac{prob(d_k = ١|R)}{prob(d_k = ٠|R)}\right] \quad (٢٤)$$

where R is the received vector. At S-th decoding iteration, sum of reliability function for all information bits is calculated and the decoder with highest reliability is selected as the chosen decoder at the transmitter. From now on, only the selected decoder continues decoding till the I-th iteration, to generate the output data by performing the hard decision on each $\tilde{L}_i^I$. A modified version of this approach is also proposed, in which a small amount of extrinsic information is present to reduce the decoding complexity. Similar scheme is proposed in [١٥٤], but instead of using internal interleavers of turbo coder, "$Q - ١$' external interleavers scramble the input data before turbo coding process. The decoder discovers the chosen interleaver by the same procedure. Note that in both methods, Q distinct IFFT blocks are required for encoding; furthermore, elimination of extrinsic information is achieved at price of increasing decoding complexity $[(Q - ١)S + I)]/I$ times. In addition, the chance of false detection of interleaver exists.

As another alternative form of candidate generation, reference [١٥٥] employs Q different time domain shifts of the input data symbols to create Q different candidates in a multiple-input-multiple-output OFDM (MIMO OFDM) system. Then, the candidate with the lowest PAPR is selected. However, no measure is suggested in [١٥٥] to recover the selected shift at the receiving end. So, $logQ$ bits of side information must also be transmitted to ensure the unambiguous recovery of the data. Obviously, the erroneous transmission of these bits will lead to erroneous recovery of the whole data.

٢) Syndrome Decoding

In standard array syndrome decoding of a linear block code, the summation of each codeword and all coset leader error patters over binary Galois Field is decoded into original codeword [١٥٦]. In the method of [١٥٧], for each block of input data, primarily the codeword is generated by a linear block code, then different candidate vectors are generated by adding the codeword to different coset leader error patterns over binary Galois Field. Then, the encoder selects the vector that produces the least PAPR and transmits it. At the receiving end, regardless of the chosen error pattern, the original codeword is selected, and decoded into the input data. Note that, in this approach, channel coding does not provide any error detection or correction, and is used solely for PAPR control. Moreover, the redundancy introduced by the coder is utilized for recovering the chosen candidate at the decoder in the absence of extrinsic information.

٣) Dummy Symbol Insertion

In dummy symbol insertion method, some input symbols do not carry any information. They are inserted among the information symbols before encoding. The values of these symbols are chosen in a way the PAPR is minimized. It is worthy to note that, this method can be considered as coding scheme equivalent to tone reservation, where some of the subcarriers are used for PAPR reduction. However, in this method, a small change in the value of dummy symbols can prompt a greater change in the resulting signal and its PAPR value, compared with tone reservation, because here a change in the value of a few input symbols can alter the codeword significantly after the encoding process.

Xin and Fair in [١٥٨] suggest insertion of dummy bits at the beginning of the data stream and then scramble data and dummy bits through division by a scrambling polynomial. By assigning different values to dummy bits, different candidates are generated and the one with the lowest PAPR is transmitted. At the receiver side, the data is descrambled and the dummy bits are discarded. Furthermore, references [١٥٩], [١٦٠] proposes inserting different label bits at the beginning of the data before feeding it to a turbo encoder. Authors of [١٦١] inserted dummy bits before an LDPC or irregular repeat accumulate (IRA) encoder, with an external interleaver for the case of systematic codes. In [١٦٢], this idea is analyzed for the general case of linear block codes with alphabet size q. The same method is also used with convolutional encoders in [١٦٣], [١٦٤].

The main disadvantage of dummy symbol insertion method is to occupy a great part of the channel capacity with symbols, which do not carry any information; to make the matter worse, this wasted capacity grows higher as the number of subcarriers increases.

٤) Puncturing Codeword

In erasure codes, the original message can be recovered from a subset of codeword symbols. Such codes are designed to achieve a reliable communication in an erasure channel. But, this property can be used for PAPR control if only a part of the codeword that creates a symbol with low PAPR is transmitted.

The idea of puncturing codeword was first offered by Fischer and Siegl [١٦٥] in ٢٠٠٩. They consider K OFDM frames as K “information symbols” that are encoded by a Reed-Solomon encoder into N “code symbols”. Then, among these code symbols, K symbols that produce the least PAPR are selected and transmitted. It can be shown that, since Reed-Solomon codes are maximum distance separable (MDS) codes, K symbols are sufficient for reconstruction of the original data. Hence, via erasure decoding [١٦٦], at the receiver, the original K symbols can be recovered and the binary data is retrieved. The only required extrinsic information is the number of frames, because the decoder must know which frames are present. Although, in this approach Reed-Solomon coding is applied, no redundancy is created, because at the same time puncturing takes place at the transmitter, and the size of coded data the same as size of the uncoded data. Hence, no error correction is provided. If error correction is also desired, more than K symbols can be transmitted, or another error correcting code, say turbo code or low-density parity check (LDPC) code can be employed across the carriers. This, in conjunction with the PAPR control code creates a product code. The authors also provided a simpler approach in which, Reed-Solomon codes are replaced by simplex codes, dual of the binary Hamming codes, and Information symbols were represented by binary vectors. Since simplex codes are not MDS codes, reconstruction of data from K vectors is not always guaranteed in the modified approach. Thus, only K linearly independent vectors can be chosen in order to ensure the recovery of the data at the receiver. This constraint specially can be harmful when low PAPR vectors are not linearly independent.

A similar idea is used in [١٦٧] by utilization of fountain codes. Fountain codes are rateless codes, originally introduced in [١٦٨], [١٦٩]. They create a potentially limitless sequence of coded symbols from a given data. In fact, the original data can be reconstructed from any portion of the coded symbols that its length is equal to or slightly greater than the length of the original data. Such codes are especially useful for broadcasting, where all receivers need to recover the original data, while each receiver might miss a part of the codeword. LT [١٧٠] or Raptor [١٧١] codes can be employed as realizations of fountain codes with good error correction performance when packet size is large. Jiang and Li [١٦٧] deploy fountain codes for PAPR control by exploiting the fact that in such codes, a portion of the codeword is enough for reconstruction of the original data. In this method, the input data packets are encoded into a sequence of fountain-coded packets that are fed into an OFDM modulator. After the generation of each symbol, the PAPR control system checks if PAPR of this symbol exceeds a pre-defined threshold or not. If this condition holds, the control system discards this symbol and generates another symbol from another portion of the coded packets, until a low PAPR symbol is found. Packet IDs must be also transmitted for reconstruction of the data at the receiver. Hence, a very small amount of extrinsic information is required too. Since packet IDs must be transmitted accurately, in the design of LT codes or Raptor codes, extra protection could be provided for the symbols that convey packet IDs. The receiver can send an optional "termination" signal after recovery of the original data or, in case of the absence of the feedback channel, the transmitter can finish transmitting when a fixed period of symbols is sent. The same idea is presented in [١٧٢] for multicast communication systems, where random fountain codes are employed for candidate generation and the signal with lowest PAPR was transmitted.

Another class of rateless codes, called spinal code [١٧٣], [١٧٤], is also exploited for PAPR reduction jointly with PTS in [١٧٥], where PAPR is reduced in two stages. First, in the coding stage, spinal code encodes the data and puncture symbols with high PAPR. Then, a normal PTS technique reduces PAPR further by phase rotation.

٥) Inverting Codeword Subsets

In this approach, similar to PTS, each codeword is divided into several disjoint partitions, each of which can be inverted to produce different candidates. But unlike conventional PTS, inversion is performed at the coding block; additionally, the decoder compensates the inversion at the receiver.

Li and Qu [١٧٦] suggest a class of low-density parity check (LDPC) codes, called PTS-LDPC codes. In such codes, each codeword is partitioned into U non-overlapping subsets, and the new codeword which also is the new candidate, is generated by inverting the value of all bits belonging to each subset. Hence, PAPR can be reduced by inverting the subsets and finding the best candidate; however, no extrinsic information about the chosen candidate and applied inversion is transmitted to the receiver: The parity check matrix is designed in a way that, no matter which subsets are inverted, the new codeword is still a valid codeword. Moreover, the bits contain extrinsic information of the inverted subsets are extracted from the received codeword as if they have been punctured or erased. Thus, the inversion is compensated at the receiver without transmission of any extrinsic information. The drawback of this method is the increased searching complexity of PTS algorithm when the number of candidates is large; additionally, wrong detection of the mentioned inversions can result in false decoding of the data.

In ٢٠١٤, invertible-subset LDPC (IS-LDPC) codes were introduced in [١٧٧]. In these codes, PAPR is reduced by inverting disjoint subsets too, with the difference of, extrinsic information of inversions being conveyed via U label bits embedded in the codeword, at highly protected positions. So that, the receiver can undo the inversion and generate the original data with higher accuracy compared to PTS-LDPC codes. Furthermore, in order to reduce searching complexity for large OFDM systems, the number of possible candidates is reduced, without sacrificing PAPR control performance. In practical OFDM systems, each codeword is transmitted by multiple OFDM symbols, say K, and PAPR of each symbol can be treated separately. If subsets are distributed uniformly among the symbols, only $٢^{U/K}$ candidates exist for each OFDM symbol. Thus, the total complexity of search for the low PAPR codeword is now $K.٢^{U/K}$ which is significantly smaller than of ٢^{U} for large values of K. In [١٧٨] the idea of IS-LDPC codes is extended to create quasi-cyclic version of IS-LDPC (IS-QC-LDPC) codes, so that the encoding process can be achieved simply by use of shift registers with linear complexity [١٧٩]. Interleaved subset mapping for IS-LDPC codes was also proposed by [١٨٠] to ensure the frequency diversity gain.

Reference [١٨١] employs the idea of inverting codeword subsets in a MIMO OFDM system. The only difference is that here subsets of the codeword are multiplied by non-negative integers instead of being inverted. Similar to other inverting codeword subset methods, the coefficients that create the lowest PAPR are selected for transmission.

TABLE I
COMPARISON OF CODING-BASED CANDIDATE GENERATION METHODS FOR PAPR CONTROL

Method	Error Correction	Extrinsic Information	Possibility of Faulty Detection of Selected Candidate
Interleaving	No	Yes	Yes
Interleaving within Turbo Encoder	Yes	No	Yes
Syndrome Decoding	No	No	No
Dummy Symbol Insertion	No	No	No
Puncturing Codeword	Optional	Yes	Yes
Inverting Codeword Subsets	Yes	Optional	Yes

IV. ANALYSIS AND COMPARISON

So far, different coding-based PAPR reduction techniques were reviewed. In this section, an analysis and a comparison between discussed techniques are provided. Generally, the covered techniques were categorized into two major classes: codes with low PAPR and coding-based candidate generation.

A. Channel Codes with Low PAPRs

The major shortcoming of codes designed to have low PAPR is the poor error correction performance especially when the number of subcarriers, and therefore the code length, increases. This is consistent with the results of [٩٦] which, shows that for each code, there is an inherent tradeoff between minimum distance, code rate, and PAPR. Thus, when parity checks are used for PAPR reduction, minimum Euclidean distance is reduced unless the code rate is decreased. As an example, in the case of Golay sequences, the problem of code rate loss deteriorates as code length increases. On the other hand, it is proved that good codes of length n with PAPR of order of $logn$ exist [٩٦], [١٤٣], [١٤٤]. So, if this value of PAPR can be tolerated, the idea of PAPR control via a channel code becomes feasible. Otherwise, other methods should be sought for this goal.

It should be noted that in the aforementioned analysis it was assumed that, coding is applied on the subcarriers within the frequency domain. In the methods that, different codes are employed in time and frequency domain, [٩٨], [٩٩], both good error control and good PAPR control can be achieved; however, the price of increased computational burden in decoding is inevitable.

B. Candidate Generation Using Channel Coding Techniques

In the case of candidate generation methods, PAPR control does not result in a degradation of error control performance; instead, when a strict PAPR control performance is desired, the problem of computational complexity deteriorates, because the generation and test of more candidates are required in order to find the one with the lowest PAPR. If system resources allow this increased computational burden, implementation of such methods in a communication system can be a good idea, otherwise, another solution must be searched for.

Channel coding techniques can be employed for candidate generation in various disciplines. In some methods, [١٥٣], [١٥٤], [١٥٩], [١٦٢]–[١٦٤], [١٧٦]–[١٧٨], [١٨٠], redundancy of the codes is also used for error correction, while in other cases, [١٤٧]–[١٤٩], [١٥٧], [١٦٥], although channel coding techniques are employed, no error correction is provided. These techniques solely control PAPR; however, in these methods, error correction can be employed separately. Within this context, both user data and parity checks can be fed to PAPR control system as the input data.

In addition to the above categorization, candidate generation methods can be classified with respect to presence of extrinsic information: In some candidate generation techniques [١٤٧]–[١٥٢], [١٦٥], [١٦٧], [١٧٢], [١٧٧], [١٧٨], extrinsic information must be transmitted to the receiver so that, the receiving end knows which candidate is selected at the transmitter. This will occupy a portion of channel capacity; however, this portion is usually relatively small. Note that, an error in transmission of the extrinsic information can result in an erroneous decoding of the whole codeword. In other cases [١٥٣], [١٥٤], [١٥٧], [١٥٩], [١٦٢]–[١٦٤], [١٧٦], decoder recovers the original data without any knowledge about the selected candidate, so no extrinsic information is required. Note that, in cases of [١٥٣], [١٥٤] this advantage is achieved at the cost of increased decoding complexity. Apart from that, incorrect detection of the selected candidate can lead to the erroneous decoding of the whole frame. Table I summarizes these classifications.

In all aforementioned candidate generation methods, the chance of high PAPR decreases as the number of candidates M increases, since the chance of PAPR exceeding a threshold δ is equal to the chance PAPR of all candidates exceeding that threshold. For the best case, when candidates are independent

$$prob(PAPR > \delta) = (١ - (١ - e^{-\delta})^{\alpha N})^{M} \qquad (٢٥)$$

where N is the size of OFDM symbol and α is the oversampling factor. When candidates are dependent, the chance of reducing PAPR is lower than (٢٥). From the computational burden point of view, increasing the number of candidates is achieved at cost of increasing computational complexity, in that M IFFT blocks are required to calculate PAPR of M different signals.

V. CONCLUSION

OFDM is an effective modulation method, which is employed in numerous practical communication systems. It has various advantages, including maximizing bandwidth efficiency and good performance in wireless channels. However, its major shortcoming is the high value of PAPR in the output signal, when the number of subcarriers is high. The mentioned peaks may exceed the linear range of PA and lead to in band and out of band distortion.

Many remedies are proposed for mitigating this problem, among which channel coding approaches have the advantage of error correction as well. In this survey, different coding-based PAPR reduction techniques are reviewed, compared and classified into two major categories: channel codes that generate low PAPR signal and candidate generation using channel coding techniques. Each category was further classified based on more subtle characteristics.

Channel codes that generate low PAPR signal generally suffer from the problem of error control performance loss. On the other hand, the major drawback of coding-based candidate generation techniques is suffering from a high computational complexity, especially when the number of subcarriers increases.

Finally, no coding-based PAPR reduction technique can provide an excellent error correction performance, a good PAPR reduction, and a low computational burden concurrently; particularly, when the number of subcarriers increases. In each system, the appropriate choice must be made based on the availability of resources and degree of tolerance for the decreased performance.

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