1) (50 points) Consider the boundary value problem:

$$-6\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial y^2} = 4 \qquad \text{over the unite square, } \Omega \text{, as shown}$$

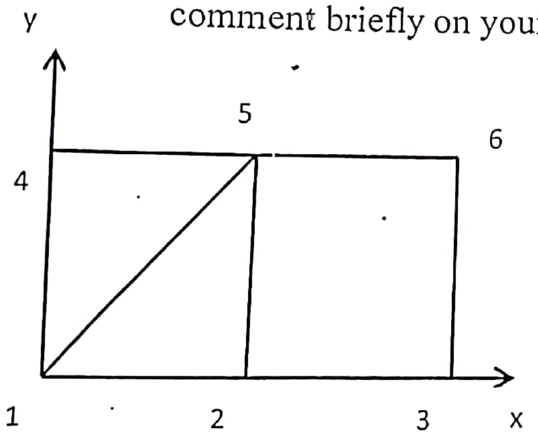
Where

$$u(0,y)=u(x,0)=2, \qquad \frac{\partial u(1,y)}{\partial x} = 0\ , \qquad \frac{\partial u(x,1)}{\partial y} = x^2 \qquad \text{on} \quad \Gamma$$

a) (25 points) Using two three-noded triangular elements and a single four-noded rectangular elements as shown, solve for the primary variables and secondary variables at grid points using the finite element methods.

b) (15 points) Solve the boundary value problem using a four-term Rayleigh Ritz polynomial approximation.

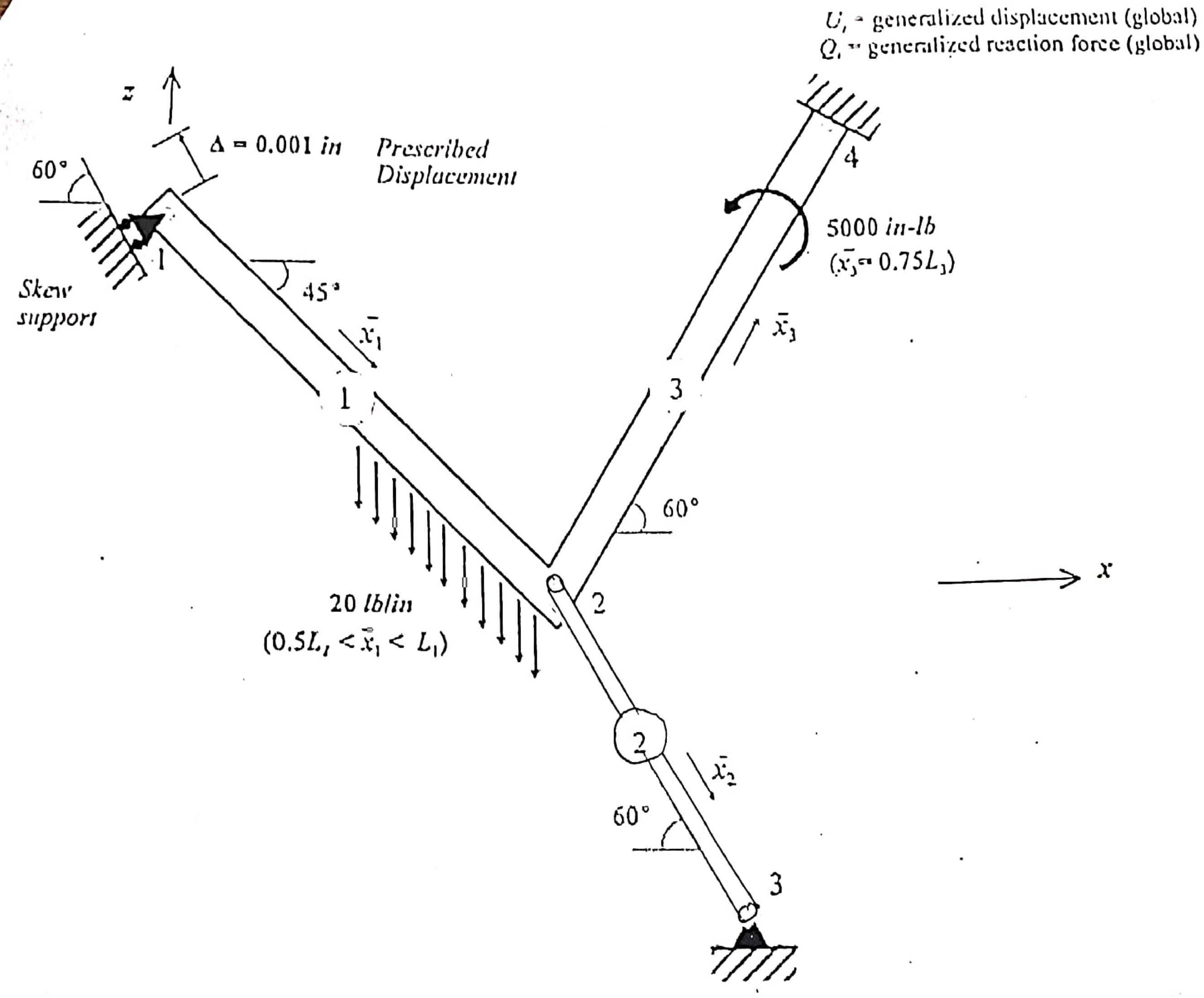
c) (10 points) Using your results from parts a) and b): Plot the primary variable and secondary along the line y=3/4 (Assume n=j). Please comment briefly on your results.



Node:	x	y
1	0	0
2	1/2	0
3	1	0
4	0	1
5	1/2	1
6	1	1

2) Consider the given plane structure defined in terms of global coordinates (x,y,z). Element 1 is welded to element 3 at global node 2. Element 2 is pinned to elements 1 and 3 at global node 2.The orientation of the local coordinate system associated with each element is specified below. The structure is subjected to mechanical loading as indicated in the figure. In addition, global node 1 has a prescribed displacement (Δ=0.001 in) measured parallel to the surface as shown in the figure.

Use the finite element methods to:



Element	E (psi)	L (in)	A (in²)	I (in⁴)
1	20·10⁶	20	6	2
2	10·10⁶	12	4	4/3
3	30·10⁶	20	6	9/2

a) Determine the element equilibrium equations for element 1 in terms of **local** coordinates. Determine the element equilibrium equations for element 1 in terms of **global** coordinates.
b) Determine the element equilibrium equations for element 2 in terms of **local** coordinates. Determine the element equilibrium equations for element 2 in terms of **global** coordinates.
c) Determine the element equilibrium equations for element 3 in terms of **local** coordinates. Determine the element equilibrium equations for element 3 in terms of **global** coordinates.
d) Determine the structural system equilibrium equations in terms of **global** coordinates, i.e., [K]{U}={F}
e) Determine the structural system equilibrium equation in terms of **skew** coordinates.
f) Solve for all generalized displacements, U_i, and forces, Q_i.