

Fig. 1. active suspension system

 (m_s)

A. System Description

The nomenclature used and parameter values, taken from [7], are given in Table I. According to the variables defined on figure 1, the equations governing the motions of sprung and unsprung mass are given by:

$$\begin{cases} m_s \ddot{z}_s + b_s (\dot{z}_s - \dot{z}_{us}) + k_s (z_s - z_{us}) = u, \\ m_{us} \ddot{z}_{us} + b_s (\dot{z}_{us} - \dot{z}_s) + k_s (z_{us} - z_s) + k_{us} (z_{us} - z_r) = -u \end{cases}$$

Choosing the set of state variables as:

$$x_1(t) = z_{us} - z_r,$$

 $x_2(t) = \dot{z}_{us},$
 $x_3(t) = z_s - z_{us},$
 $x_4(t) = \dot{z}_s$

and $w=\dot{z}_r$ (ground vertical velocity), the state space description of the system is obtained as:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_{us}/m_{us} & -b_s/m_{us} & k_s/m_{us} & b_s/m_{us} \\ 0 & -1 & 0 & 1 \\ 0 & b_s/m_s & -k_s/m_s & -b_s/m_s \end{bmatrix}$$

$$B_1^T = \begin{bmatrix} -1 & 0 & 0 & 0 \end{bmatrix}$$

$$B_2^T \stackrel{!}{=} \begin{bmatrix} 0 & -u_s/m_{us} & 0 & u_s/m_s \end{bmatrix}$$

where the control input u is defined as u_f/u_s , with u_s being the normalizing factor and u_f actuator real force. The body (sprung) mass usually changes with the vehicle load. It is considered that sprung mass has a 20% change ($\approx 64kg$) around its nominal value.

B. Derivation of design framework

In the next sections we will design both static and dynamic output feedback H_{∞} controller for the system described above. To provide equal conditions for comparison of both strategies, identical framework (weights) is used for the designs. Therefore in the following we cast the problem into standard H_{∞} framework, which will be used for both

Model parameters	symbol	values	unit
sprung mass	m_s	320 ± 64	kg
suspension stiffness	k_s	18000	N/m
suspension damping rate	b_s	1000	N/(m/sec)
Wheel assembly mass	m_{us}	40	kg ,
tire stiffness	k_{us}	200000	N/m
normalizing factor	u_s	1500	N

TABLE I

Nomenclature and nominal values of the parameters in a Quarter car model [7]

designs.

In designing the control law for a suspension system, the following requirements are taken into consideration:

1. Ride Comfort: Ride comfort of a vehicle, also known as vibration isolation ability, is judged by the RMS value of the acceleration, sensed by vehicle passengers. To design a system to perform satisfactorily for a wide range road irregularities, and considering the fact that there are various uncertainties in the system, calls for minimizing the H_{∞} norm of the transfer function from road disturbance to body acceleration. Recall that H_{∞} norm of a system is its worst case output energy (RMS value). But the fact that sprung mass resonance frequency occures around 1 Hz and according to ISO-2631 human body is more sensitive to frequencies near 4-8 Hz in the vertical direction [14], requires more stress on minimizing the acceleration in these frequencies. Hence the weighted body acceleration is chosen as the first controlled output, i.e.,

$$z_1 = W_{z1}\ddot{z}_s$$

where

$$W_{z1} = \frac{0.8(0.9s^2 + 160s + 35)}{s^2 + 20s + 100}$$

This weight has its peak in the frequency range 0.3-10 Hz to emphasize the importance of minimization in this frequency range.

2. Ride safety: Firm uninterrupted contact of wheels to road against road disturbances (good road holding) is necessary for vehicle handling and leads to ride safety. Therefore to ensure ride safety, it is required that the transfer function from road disturbance to tyre deflection, $z_{us} - z_r$, is kept small. To achieve this goal for all kinds of disturbances, H_{∞} norm is chosen as performance measure. Hence the second controlled output is considered as the ratio of dynamic-to-static tyre load:

$$z_2(t) = \frac{k_{us}(z_{us} - z_r)}{(m_s + m_{us})g}$$

To keep controller degree as small as possible, this output is minimized over all frequencies by adopting a weighting function of 1. However it is pointed out that one can consider a frequency-dependent weighting function with emphasis on minimization of the output at wheel-hop frequency (11.254 Hz).

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3. Damage minimization: Due to mechanical structure constraints, it is important to prevent excessive suspension bottoming. Hitting the deflection limit not only results in deterioration of ride comfort, but also even may cause structural damage. Thus it is important to keep H_{∞} -norm of the transfer function from road disturbance to suspension deflection small. It implies to choose:

$$z_3(t) = z_s - z_{us} = x_3(t)$$

Since suspension deflection is frequency-independent, we simply choose unity weight for it.

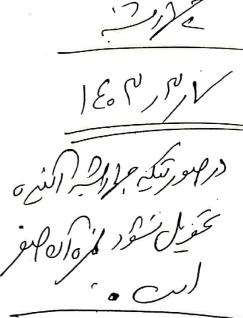
4. Actuator limits: Considering limited power and bandwidth of the actuator, we define:

$$z_4(t) = W_u u$$

where W_u is a first order lead term with corner frequencies of 15 Hz and 150 Hz. The magnitude of weight increases above 15 Hz to emphasize on the importance of minimization of control signal at higher frequencies.

Thus the H_{∞} control problem is formulated as in Figure 2, where $z=\begin{pmatrix} z_1 & z_2 & z_3 & z_4 \end{pmatrix}^T$. It is also assumed that suspension deflection (z_s-z_{us}) and vertical velocity of sprung mass (\dot{z}_s) are the measured variables (y).

Now the objective is to find a controller K such that the H_{∞} -norm of closed loop transfer function, T_{zw} , is minimized.



Augmented Plant

$$x = Ax + B, w + B, u$$
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 $x = Ax + B, w + B, u$
 $x = C_1x + D_1w + C_{12}u$

Fig. 2. design framework

 $x = Ax + B, w + B, u$
 $x = C_1x + D_1w + C_{12}u$

 $Z_{r} = \begin{cases} A/2(1-cos \frac{2\pi Vt}{L}) \end{cases}$

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A = 0.05m L = 5m $V = 20 \frac{m}{8}$ $V = 20 \frac{m}{8}$