Advanced Production Decline Analysis (APDA)

Well Performance

Shahab Gerami

Outline

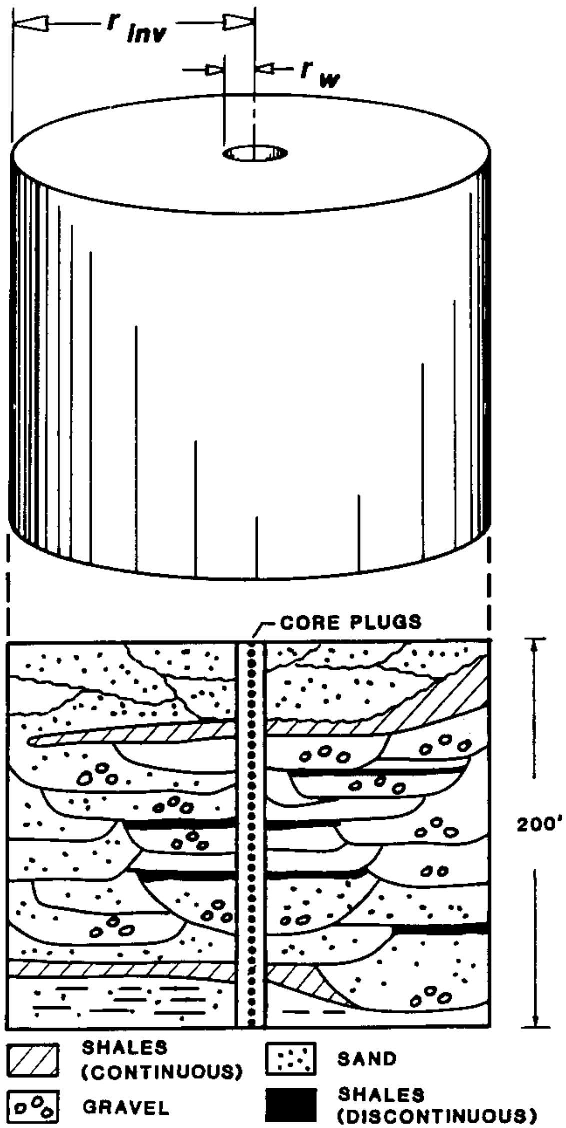
- **An Introduction to Well Performance Analysis**
  - Flow regimes,
  - Basic well model
  - Simplifying assumptions
  - Diffusivity equation for slightly compressible oil,
  - Solution to diffusivity equation,
    - Constant rate solution
    - Constant pressure solution
  - Application of the Solution (Type curves)
  - Treatment of the Diffusivity Equation for Gas reservoirs

Development of Hydraulic Diffusivity Equation
1-D, Radial, Single Phase, Slightly Compressible

- Physical model
- Simplifying assumptions
- Mathematical model
  - Choosing an appropriate element
  - Governing equation
    - Mass balance
    - Momentum balance (Darcy's law)
    - Equation of state
  - Initial and Boundary conditions
    - Infinite acting
      - Constant rate production
      - Constant pressure production
    - Finite acting
      - Constant rate production
      - Constant pressure production
  - Solutions
    - Laplace space solutions
    - Time domain solutions
    - Simplified solutions
- Applications (Drawdown (single rate & multi rate), Reservoir limit test, Build up, Superposition (time & space), …),

3

Physical Model



← Reservoir Engineering Model

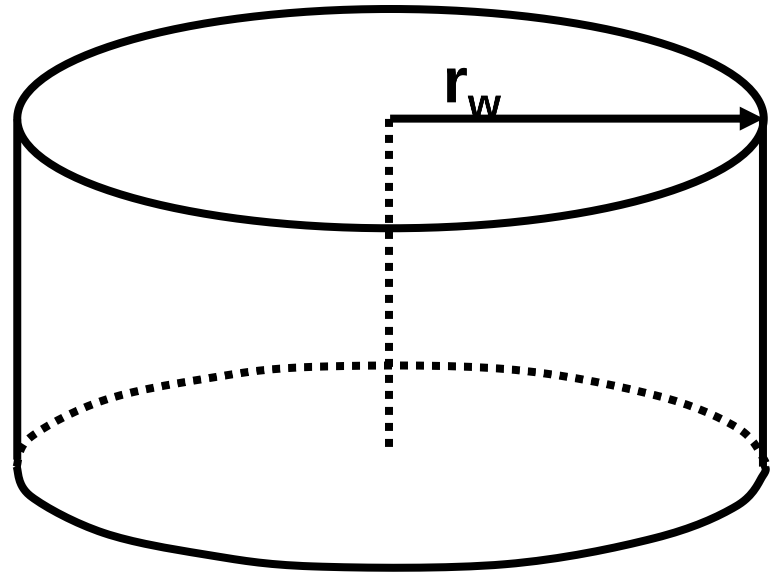
- Works 95+ percent of the time...
- Why? Pressure and volume averaging of reservoir properties.
- When does it not work? High contrast in reservoir properties.

← Actual Reservoir Model

- Complex geology.
- Complex fluid behavior.
- Poor lateral (and vertical) continuity.

Simplifying Assumptions

- Homogeneous
- Isotropic
- Ignore Gravity
- Constant Temperature
- Darcy's law applies
- Single phase fluid
- Radial flow
- Totally penetrating vertical well
- Constant net pay, saturation
- $(\partial p/\partial r)$ - gradient in reservoir - is small
- Constant wellbore storage
- Constant pressure throughout reservoir at time t = 0
- Constant production rate
- Closed circular reservoir
- Model complexities will be introduced as necessary



<u>OIL/WATER</u>

- Compressibility is small and constant
- Viscosity is constant
- Laminar flow

Mathematical Model-Governing Equation

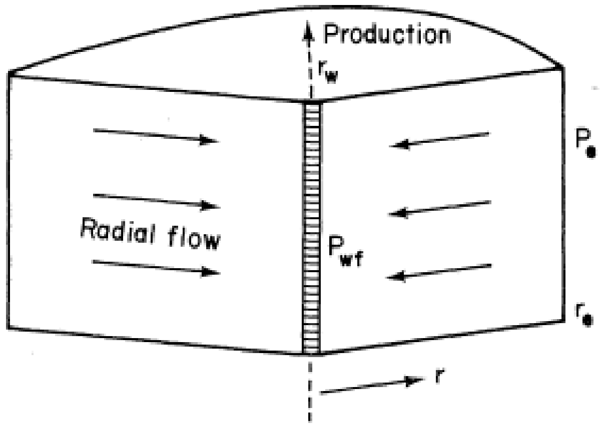
- **Mass balance**

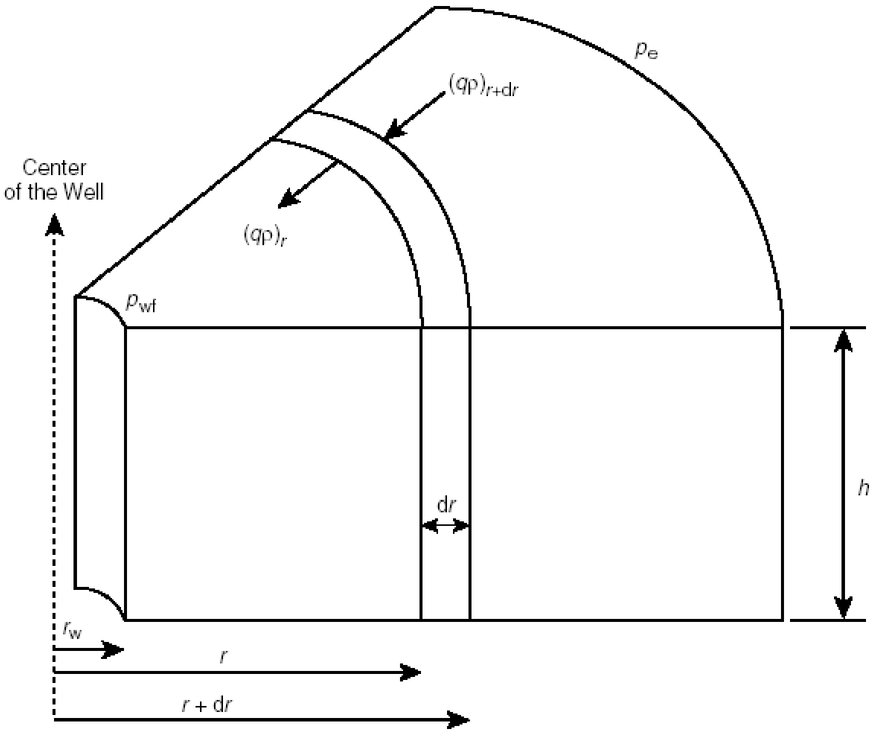
$$\left(-\rho_o A v\right)_{r+\Delta r} - \left(-\rho_o A v\right)_r = \frac{\left(\rho_o \Delta V\right)_{t+\Delta t} - \left(\rho_o \Delta V\right)_t}{\Delta t}$$

- **Momentum balance (Darcy's law)**

$$v_{gr} = -\frac{k}{\mu}\frac{\partial p}{\partial r}$$

$$\rho_o = \rho_{ob} \exp\left(c_o\left(p - p_b\right)\right)$$





Hydraulic Diffusivity Equation

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]=\frac{1}{\eta}\frac{\partial p}{\partial t}$$

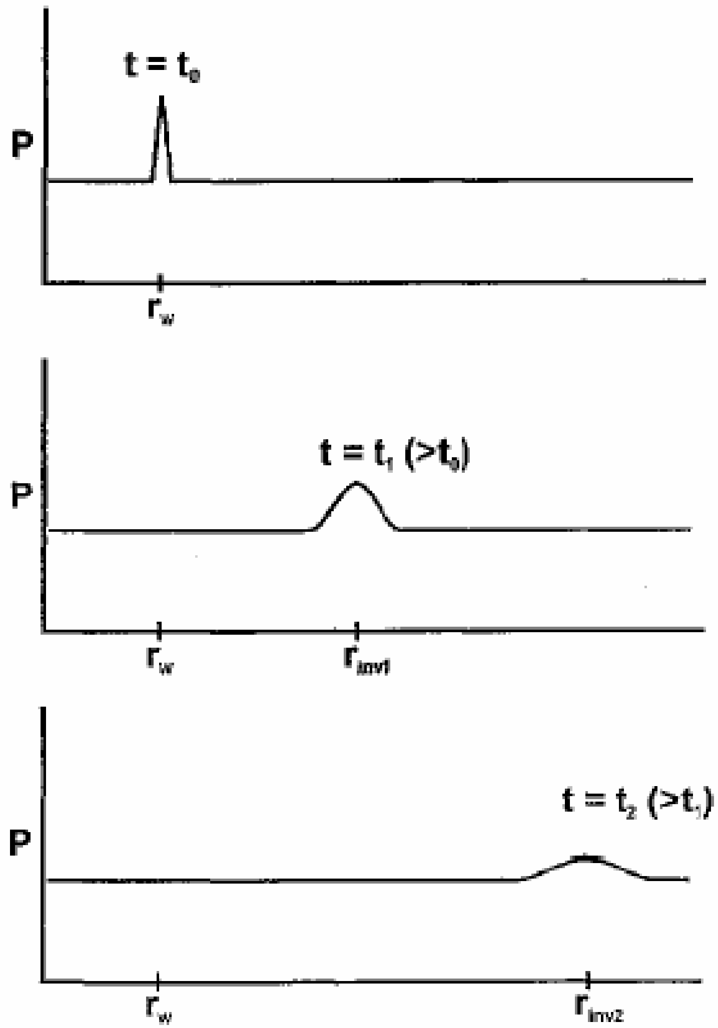
$$\eta=\frac{k}{\phi\mu\, c_t}$$

Hydraulic diffusivity equation determines the velocity at which pressure waves propagate in the reservoir. The more the permeability the faster the pressure wave will propagate.

7

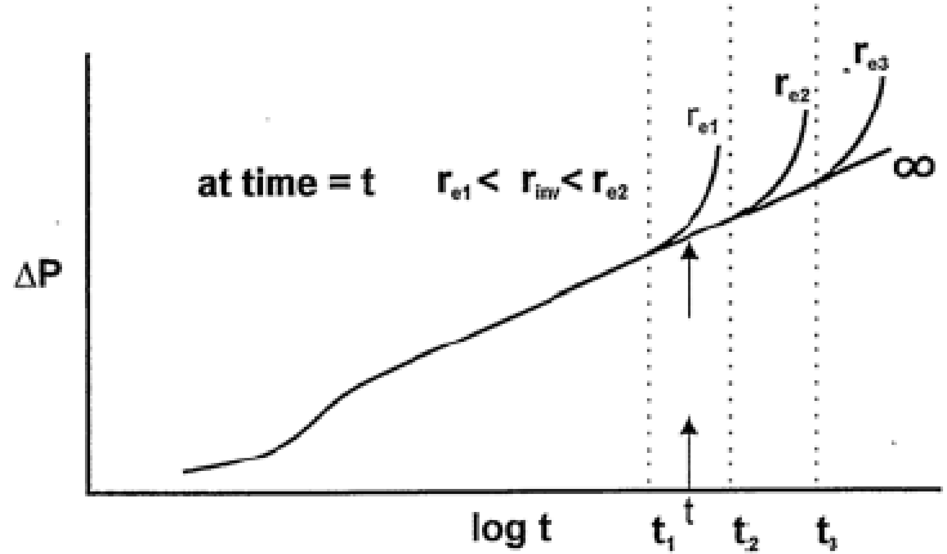
Travel of an impulse to illustrate RADIUS of INVESTIGATION

$$r_{inv} \approx 0.0325\sqrt{\left(\frac{kt}{\phi\mu c}\right)}$$



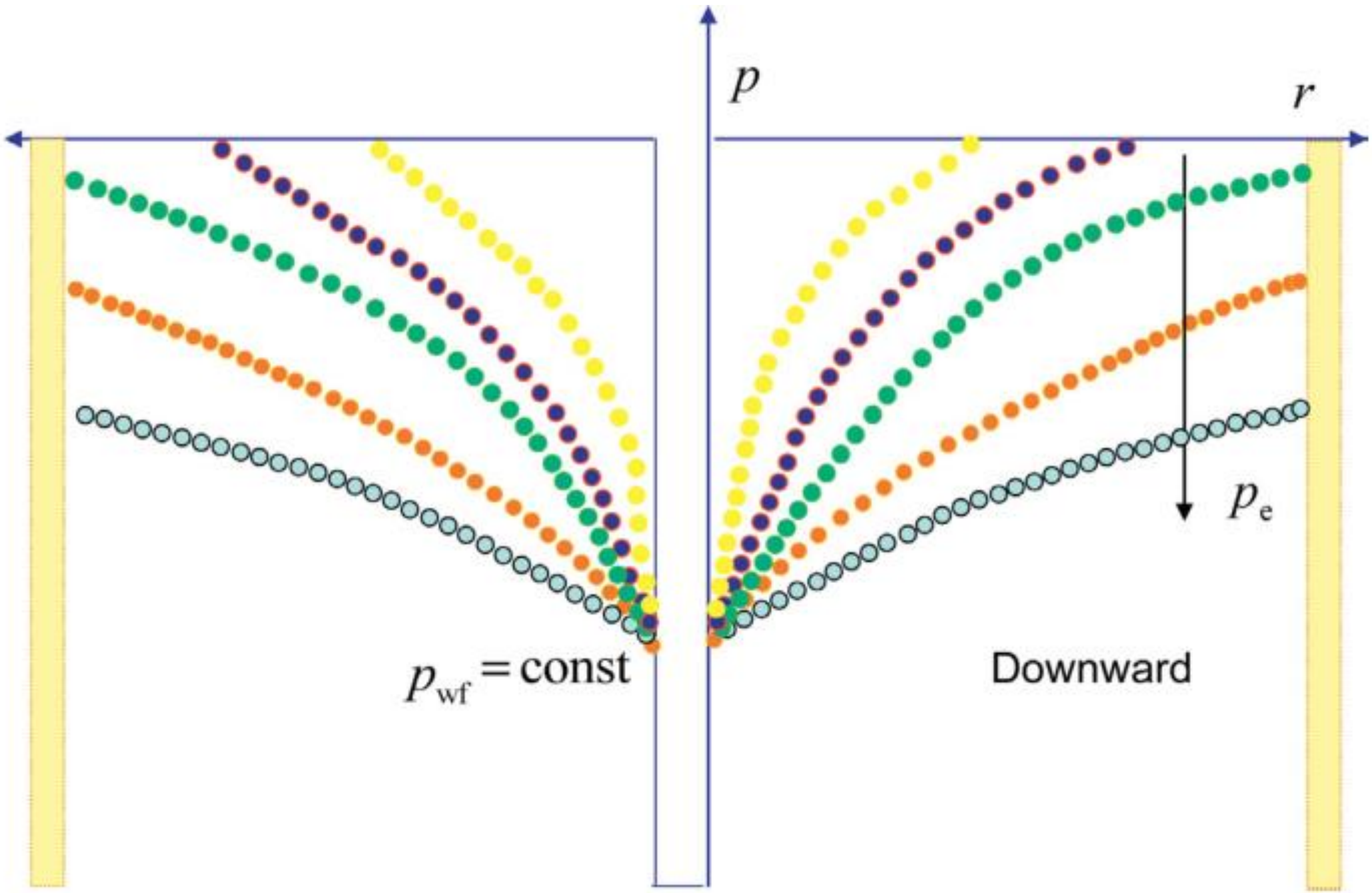
Radius of Investigation

- How far into the reservoir have we investigated
- Dimensionless curves deviate from radial flow (semi-log straight line) at $t_D = 1/4\ re_D^2$
- $r_{inv} \approx 0.0325 \sqrt{(kt)/(\phi\mu c)}$
- Independent of rate
- Approximate
- Total reservoir affected even at small times
- Practical concepts
- NOT Theoretical
- Used to calculate approximate distance to boundary

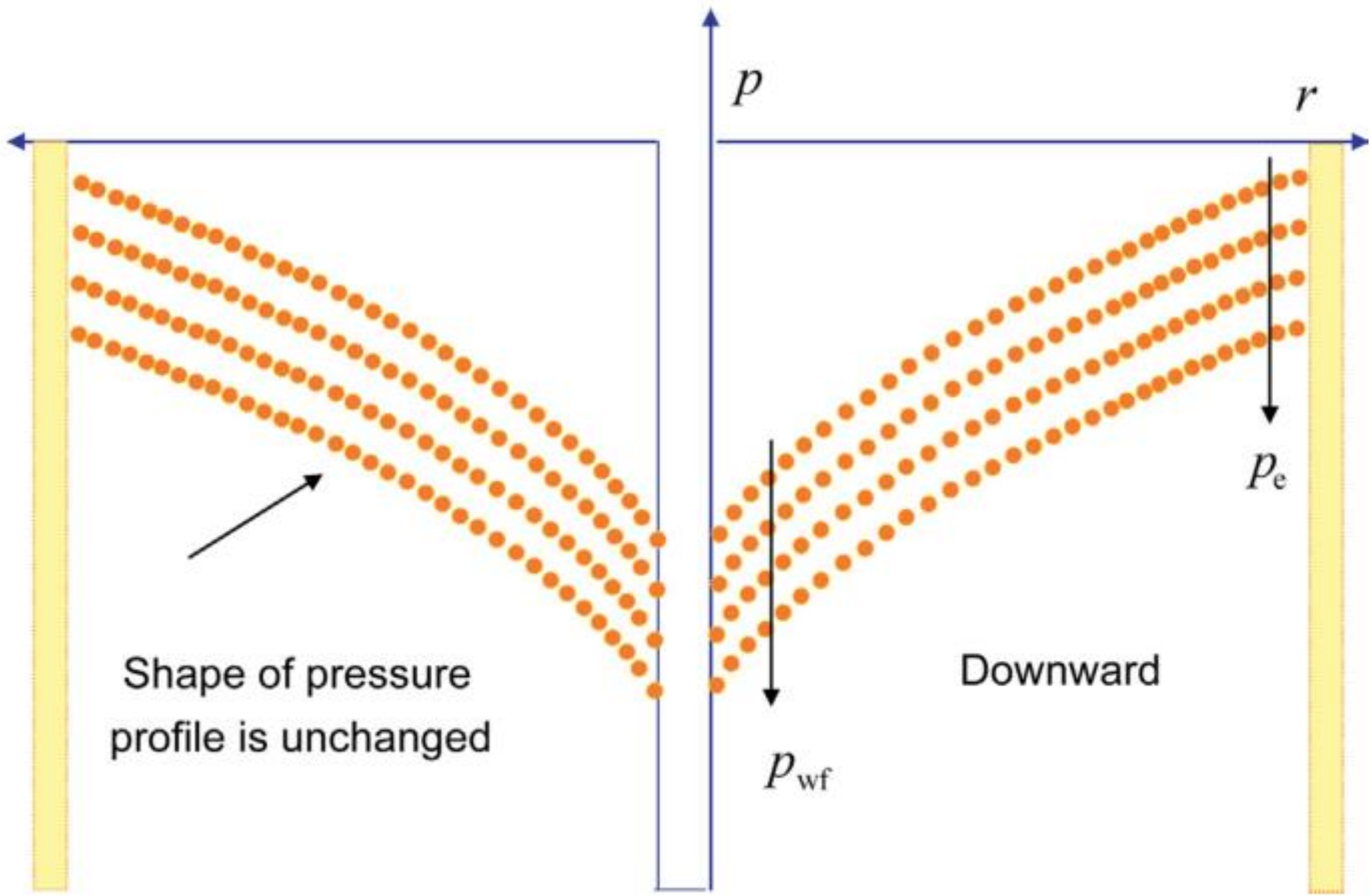


$$r_i = \sqrt{\frac{kt}{948\phi\mu C_t}}$$

Pressure Distribution During Unsteady State Flow



Pressure Distribution During Pseudo Steady State Flow



Mathematical Model-Governing Equation

A reservoir model is the superposition of reservoir, inner, and outer boundary conditions

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]=\frac{\varphi\mu c_t}{k}\frac{\partial p}{\partial t}$$

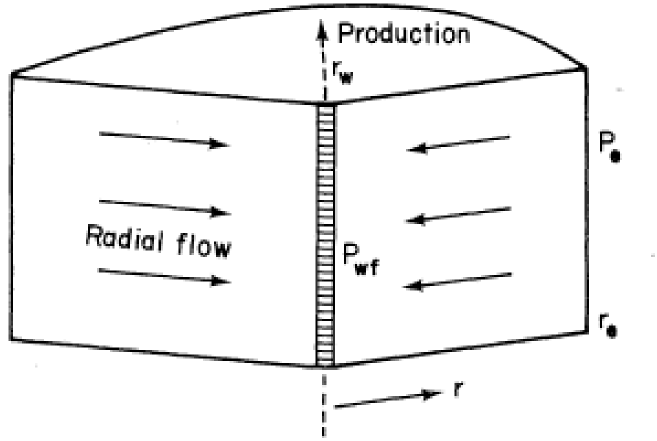


Fig. 9.1 Radial flow towards a well.

$Initial \quad Condition: \quad p = p_i, \qquad t = 0, \quad r \ge r_w$

Well production	Flow regime	Inner Boundary Condition	Outer Boundary conditions
Constant rate	Finite acting (Bounded)	$\left(\frac{\partial p}{\partial r}\right)_{r_w} = -\frac{\mu q B_o}{2\pi r_w h k}$	$\left(\frac{\partial p}{\partial r}\right)_{r\to re} = 0$
Constant pressure	Finite acting (Bounded)	$(p)_{r_w} = p_{wf}$	$\left(\frac{\partial p}{\partial r}\right)_{r\to re} = 0$

Dimensionless Variables

The advantages of using a dimensionless physical quantity:

- ❑ The results are suitable for different unit systems.
- ❑ The number of parameters and variables are reduced.
- ❑ The problem is simplified.
- ❑ The nature of physical problem is better shown.

$$r_D = \frac{r}{r_w} \quad r_{eD} = \frac{r_e}{r_w} \quad t_D = \frac{\alpha_t K t}{\varphi \mu C_t r_w^2} \quad t_{DA} = \frac{\alpha_t K t}{\varphi \mu C_t A}$$

$$p_D = \frac{Kh}{\alpha_p q \mu B}(p_i - p) \quad p_{wD} = \frac{Kh}{\alpha_p q \mu B}(p_i - p_{wf}) \quad C_D = \frac{\alpha_c C}{\varphi C_t h r_w^2}$$

Different Systems of Units

Unit system		Basic SI	Legal SI	Imperial field unit
Length	***r*, *h*, *L***	**m**	**m**	**ft**
Time	t	s	h	h
Pressure	p	Pa	MPa	psi
Permeability	K	m^2	mD	md
Oil Rate	q_o	m^3/s	m^3/d	STB/D
Gas Rate	q_g	m^3/s	$10^4 m^3/d$	Mscf/D
Viscosity	μ	Pa·s	mPa·s(= cP)	cP
Conversion	α_p	$1/2\pi$	1.842	141.2
factor	α_t	1	0.0036	2.637×10^{-4}
	α_c	$1/2\pi$	$1/2\pi$	0.8936
	α_m	$1/\pi$	3.683×10^4	50312

$$r_D = \frac{r}{r_w} \quad r_{eD} = \frac{r_e}{r_w} \quad t_D = \frac{\alpha_t K t}{\varphi \mu C_t r_w^2} \quad t_{DA} = \frac{\alpha_t K t}{\varphi \mu C_t A}$$

$$p_D = \frac{Kh}{\alpha_p q \mu B}(p_i - p) \quad p_{wD} = \frac{Kh}{\alpha_p q \mu B}(p_i - p_{wf}) \quad C_D = \frac{\alpha_c C}{\varphi C_t h r_w^2}$$

Dimensionless Variables

$$p = p_i + 70.6\frac{qB\mu}{kh}Ei\left(-\frac{948\phi\mu c_t r^2}{kt}\right)$$

$$\boxed{\frac{kh(p_i - p)}{141.2qB\mu}} = -\frac{1}{2}Ei\left(-\frac{\left(\boxed{\frac{r}{r_w}}\right)^2}{4\left(\boxed{\frac{0.0002637kt}{\phi\mu c_t r_w^2}}\right)}\right)$$

$$r_D \equiv \frac{r}{r_w}$$

$$p_D \equiv \frac{kh(p_i - p)}{141.2qB\mu}$$

$$t_D \equiv \frac{0.0002637kt}{\phi\mu c_t r_w^2}$$

$$p_D = -\frac{1}{2}Ei\left(-\frac{r_D^2}{4t_D}\right)$$

Dimensionless Hydraulic Diffusivity Equation

$$\boxed{\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]=\frac{\varphi\mu c_t}{k}\frac{\partial p}{\partial t}}$$

$$p_D=\frac{kh\Delta p}{141.2\ qB\mu}, \tag{1-2}$$

$$t_D=\frac{0.000264\ kt}{\phi\mu c_t r_w^2}, \text{ and} \tag{1-3}$$

$$r_D=r/r_w \tag{1-4}$$

$$\frac{\partial^2 p_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_D}{\partial r_D}=\frac{\partial p_D}{\partial t_D}$$

Pressure Distribution in a Closed Circular Reservoir - Constant Rate Production

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)=\frac{\varphi\mu C_{\mathrm{t}}}{K}\frac{\partial p}{\partial t} \qquad (4.1)$$

$$p(r,0)=p_{\mathrm{i}} \qquad (4.2)$$

$$\left(r\frac{\partial p}{\partial r}\right)_{r=r_{\mathrm{w}}}=\frac{q\mu B}{2\pi Kh} \qquad (4.3)$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_{\mathrm{e}}}=0 \qquad (4.4)$$

In dimensionless variables

$$\frac{1}{r_{\mathrm{D}}}\frac{\partial}{\partial r_{\mathrm{D}}}\left(r_{\mathrm{D}}\frac{\partial p_{\mathrm{D}}}{\partial r_{\mathrm{D}}}\right)=\frac{\partial p_{\mathrm{D}}}{\partial t_{\mathrm{D}}} \tag{4.5}$$

$$p_{\mathrm{D}}\left(r_{\mathrm{D}},0\right)=0 \tag{4.6}$$

$$\left(r_{\mathrm{D}}\frac{\partial p_{\mathrm{D}}}{\partial r_{\mathrm{D}}}\right)_{r_{\mathrm{D}}=1}=-1 \tag{4.7}$$

$$\left.\frac{\partial p_{\mathrm{D}}}{\partial r_{\mathrm{D}}}\right|_{r_{\mathrm{D}}=r_{\mathrm{eD}}}=0 \tag{4.8}$$

The dimensionless variables are defined as follows

$$p_{\mathrm{D}}=\frac{2\pi Kh\left(p_{\mathrm{i}}-p_{\mathrm{wf}}\right)}{q\mu B} \quad t_{\mathrm{D}}=\frac{Kt}{\varphi\mu C_{\mathrm{t}}r_{\mathrm{w}}^{2}} \quad r_{\mathrm{D}}=\frac{r}{r_{\mathrm{w}}} \quad r_{\mathrm{eD}}=\frac{r_{\mathrm{e}}}{r_{\mathrm{w}}}$$

Laplace transformation of Eq. (4.5) through Eq. (4.8) yields

$$\frac{d^2 \bar{p}_{\mathrm{D}}}{dr_{\mathrm{D}}^2} + \frac{1}{r_{\mathrm{D}}} \frac{d \bar{p}_{\mathrm{D}}}{dr_{\mathrm{D}}} = s \bar{p}_{\mathrm{D}} \tag{4.9}$$

$$\left. \frac{d \bar{p}_{\mathrm{D}}}{dr_{\mathrm{D}}} \right|_{r_{\mathrm{D}}=1} = -\frac{1}{s} \tag{4.10}$$

$$\left. \frac{d \bar{p}_{\mathrm{D}}}{dr_{\mathrm{D}}} \right|_{r_{\mathrm{D}}=r_{\mathrm{eD}}} = 0 \tag{4.11}$$

In the Laplace space, the solution of Eq. (4.9) is given as follows

$$\bar{p}_{\mathrm{D}} = A_0 I_0 \left(r_{\mathrm{D}} \sqrt{s} \right) + B_0 K_0 \left(r_{\mathrm{D}} \sqrt{s} \right) \tag{4.12}$$

$$\bar{p}_D = A_0 I_0\left(r_D\sqrt{s}\right) + B_0 K_0\left(r_D\sqrt{s}\right) \tag{4.12}$$

According to the inner boundary condition Eq. (4.10), we have

$$A_0 I_1\left(\sqrt{s}\right) - B_0 K_1\left(\sqrt{s}\right) = -\frac{1}{s\sqrt{s}} \tag{4.13}$$

According to the outer boundary condition Eq. (4.11), we have

$$A_0 I_1\left(r_{eD}\sqrt{s}\right) - B_0 K_1\left(r_{eD}\sqrt{s}\right) = 0 \tag{4.14}$$

According to Eq. (4.13) and Eq. (4.14), we have

$$A_0 = \frac{\dfrac{1}{s\sqrt{s}} \dfrac{K_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}{I_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}}{K_1\left(\sqrt{s}\right) - I_1\left(\sqrt{s}\right)\dfrac{K_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}{I_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}} \tag{4.15}$$

$$B_0 = \frac{\dfrac{1}{s\sqrt{s}}}{K_1\left(\sqrt{s}\right) - I_1\left(\sqrt{s}\right)\dfrac{K_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}{I_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}} \tag{4.16}$$

The definition of pressure derivative is as follows

$$p_{D}' = \frac{dp_{D}}{dt_{D}} \tag{4.18}$$

The following expression is obtained through the Laplace transformation of Eq. (4.18)

$$\bar{p_{D}'} = s\,\bar{p}_{D} = \frac{1}{\sqrt{s}}\left[\frac{\dfrac{K_1\left(r_{eD}\sqrt{s}\right)}{I_1\left(r_{eD}\sqrt{s}\right)} + \dfrac{K_0\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)}}{\dfrac{K_1\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)} - \dfrac{I_1\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)}\dfrac{K_1\left(r_{eD}\sqrt{s}\right)}{I_1\left(r_{eD}\sqrt{s}\right)}}\right] \tag{4.19}$$

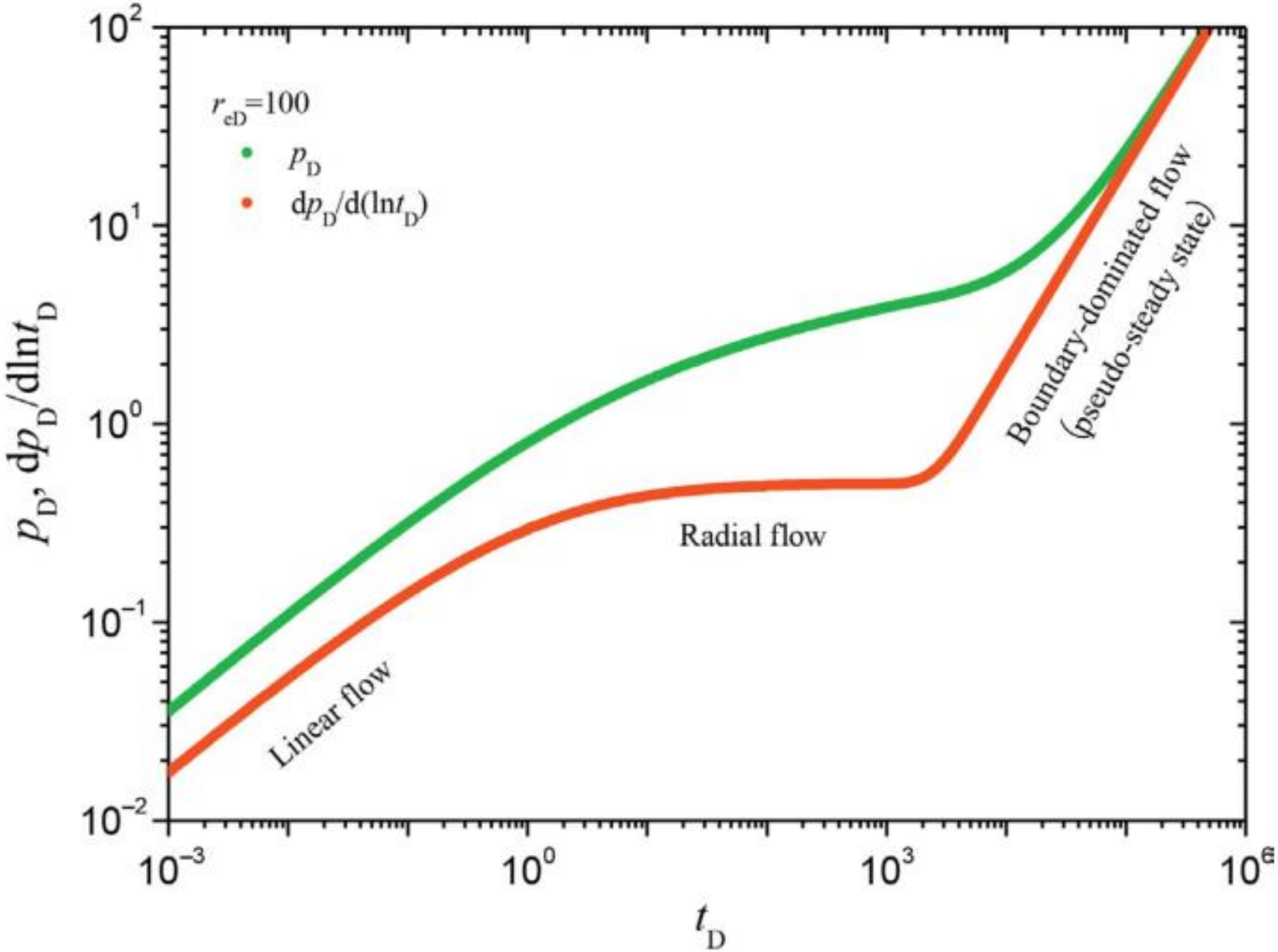


Figure 4.1 Pressure distribution in a closed circular reservoir: constant rate production

Before the late period, Eq. (4.17) can be simplified as

$$\bar{p}_{\mathrm{D}} = \frac{1}{s\sqrt{s}} \frac{K_0\left(\sqrt{s}\right)}{K_1\left(\sqrt{s}\right)} \tag{4.20}$$

In the early period, when $t_{\mathrm{D}} < 0.01$, that is, the big s period, we have

$$K_v(s) = \sqrt{\frac{\pi}{2s}}\, e^{-s} \tag{4.21}$$

Substituting the asymptotic formula Eq. (4.21) into Eq. (4.20), we have

$$\bar{p}_{\mathrm{D}} = \frac{1}{s\sqrt{s}} \tag{4.22}$$

According to the mathematics handbook, there is

$$L^{-1}\left[\frac{1}{s\sqrt{s}}\right] = 2\sqrt{\frac{t_{\mathrm{D}}}{\pi}} \tag{4.23}$$

Performing the Laplace inversion transformation of Eq. (4.22), we have

$$p_D = 2\sqrt{t_D/\pi} \tag{4.24}$$

Eq. (4.24) reflects the linear flow characteristic in the early period.
In the big time period, $t_D/r_D^2 > 100$, we have

$$K_0(s) \approx -\left(\ln\frac{s}{2} + \gamma\right) \tag{4.25}$$

$$K_1(s) \approx \frac{1}{s} \tag{4.26}$$

Substituting Eq. (4.25) and Eq. (4.26) into Eq. (4.20), we have

$$\bar{p}_D = \frac{-\left(\ln\frac{\sqrt{s}}{2} + \gamma\right)}{s} = -\frac{\ln s}{2s} - \frac{\gamma - \ln 2}{s} \tag{4.27}$$

According to the mathematics handbook, there is

$$L^{-1}\left[\frac{\ln s}{s}\right]=-\ln t_{\mathrm{D}}-\gamma \tag{4.28}$$

Performing the Laplace inversion transformation of Eq. (4.27), we have

$$p_{\mathrm{D}}=\frac{1}{2}\left(\ln t_{\mathrm{D}}+\gamma\right)-\left(\gamma-\ln 2\right)=\frac{1}{2}\ln\frac{4t_{\mathrm{D}}}{e^{\gamma}} \tag{4.29}$$

Eq. (4.29) reflects the radial flow characteristic in the big time period.

Van Everdingen and Hurst (1949) presented the analytical solution of Eq. (4.17) in Euclidean space through Laplace transformation

$$\begin{aligned} p_{\mathrm{D}}(r_{\mathrm{D}},t_{\mathrm{D}})=&\frac{2}{(r_{\mathrm{eD}}^2-1)}\left(t_{\mathrm{D}}+\frac{r_{\mathrm{D}}^2}{4}\right)-\frac{r_{\mathrm{eD}}^2\ln r_{\mathrm{D}}}{(r_{\mathrm{eD}}^2-1)}-\frac{3r_{\mathrm{eD}}^4-4r_{\mathrm{eD}}^4\ln r_{\mathrm{eD}}-2r_{\mathrm{eD}}^2-1}{4(r_{\mathrm{eD}}^2-1)^2}\\ &+\pi\sum_{n=1}^{\infty}e^{-z_n^2t_{\mathrm{D}}}\frac{J_1^2(z_nr_{\mathrm{eD}})\left[J_1(z_n)Y_0(z_nr_{\mathrm{D}})-J_0(z_nr_{\mathrm{D}})Y_1(z_n)\right]}{z_n\left[J_1^2(z_nr_{\mathrm{eD}})-J_1^2(z_n)\right]} \end{aligned} \tag{4.30}$$

where Z_n is the root of the following characteristic equation

$$Y_1(z_n)J_1(z_n r_{\mathrm{eD}}) - J_1(z_n)Y_1(z_n r_{\mathrm{eD}}) = 0 \tag{4.31}$$

Under the pseudo-steady state, Eq. (4.30) can be simplified as

$$p_{\mathrm{D}} \approx \frac{2t_{\mathrm{D}}}{r_{\mathrm{eD}}^2 - 1} + \ln r_{\mathrm{eD}} - \frac{3}{4} \tag{4.32}$$

van Everdingen- Hurst Constant Terminal Rate Solution
Bounded Cylindrical Reservoir
(exact solution)

$$p_{wD}(t_D) = \frac{2t_D}{r_{eD}^2} + \ell n(r_{eD}) - 0.75 + 2\sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 t_D} J_1^2(\alpha_n r_{eD})}{\alpha_n^2 \left[J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)\right]}$$

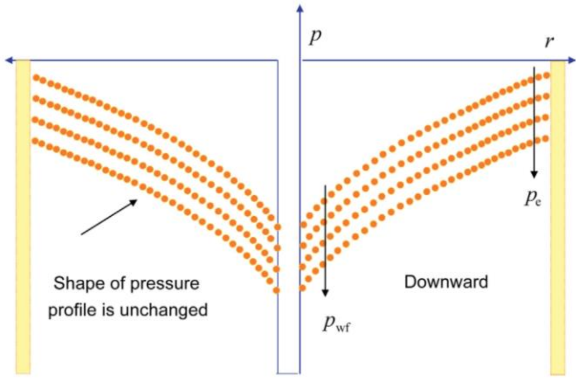
$$J_1(\alpha_n r_{eD})Y_1(\alpha_n) - J_1(\alpha_n)Y_1(\alpha_n r_{eD}) = 0 \Rightarrow \alpha_n$$

Approximate Solutions

1. Infinite cylindrical reservoir with line-source well
2. Bounded cylindrical reservoir, pseudo steady-state flow

Solutions- Laplace Domain (Sabet, 1991).

Constant rate solution	**Infinite-acting reservoir** $$p_{wD}(t_D) = \frac{1}{2}\left[\ell n(t_D) + 0.80908\right]$$	
	Boundary dominated flow- approximate late time $$p_{wD}(t_D) = \frac{2t_D}{r_{eD}^2} + \ell n(r_{eD}) - 0.75$$	



29

Van Everdingen and Hurst (1949) pointed out that according to the superposition principle, the relationship between the pressure solution at a constant production rate and the production rate solution under constant pressure turned out to be

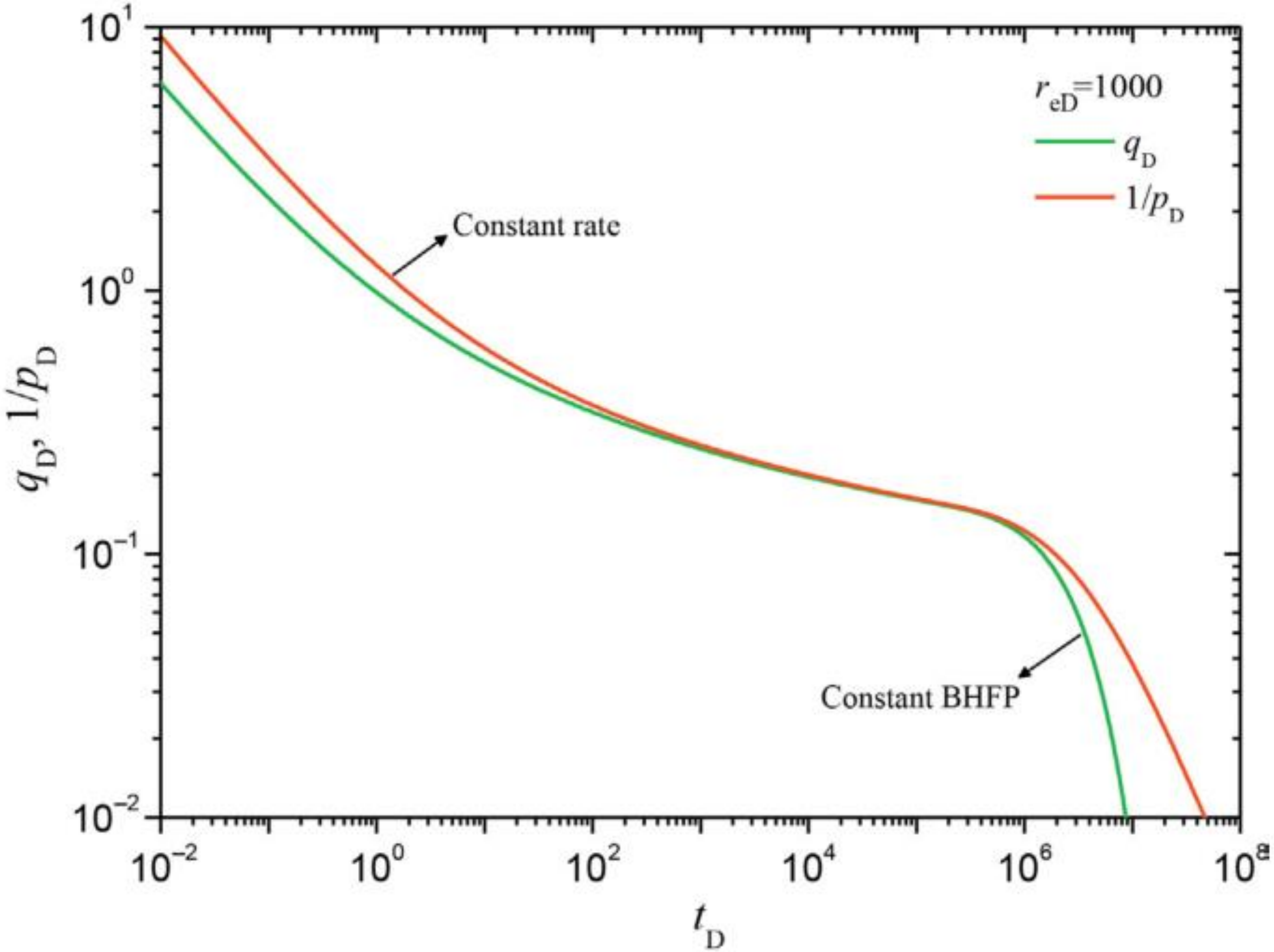
$$\bar{p}_{\mathrm{D}}(s)\bar{q}_{\mathrm{D}}(s)=\frac{1}{s^2} \tag{4.33}$$

Substituting Eq. (4.17) into Eq. (4.33), we have

$$\bar{q}_{\mathrm{D}}=\frac{\dfrac{K_1\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)}-\dfrac{I_1\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)}\dfrac{K_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}{I_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}}{\sqrt{s}\left[\dfrac{K_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}{I_1\left(r_{\mathrm{eD}}\sqrt{s}\right)}+\dfrac{K_0\left(\sqrt{s}\right)}{I_0\left(\sqrt{s}\right)}\right]} \tag{4.34}$$

It is the production rate equation Eq. (3.22) under constant BHFP.

Plotting q_D (the production solution under constant BHFP) and $1/p_D$ (the reciprocal of p_D, which is the pressure solution at a constant rate) in the same coordinate system, as shown in Figure 4.2. The figure shows that in the transient flow period, the two curves are almost overlapped and dispersed in the boundary-dominated flow period. In this period, the q_D~t_D curve follows a exponential decline trend while the $1/(p_D–t_D)$ curve exhibits a harmonic decline trend.



Reservoir-Limits Test
(Estimation of Reservoir Pore Volume)

$$p_{wf} = p_i - \frac{141.2 qB\mu}{kh}\left[\frac{0.0005274k}{\phi\mu\, c_t r_e^2} t + \ell n\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

$$\frac{\partial p_{wf}}{\partial t} = \frac{0.07447 qB_o}{\phi\, c_t r_e^2}$$

$$V_p = \pi\, r_e^2 h\phi$$

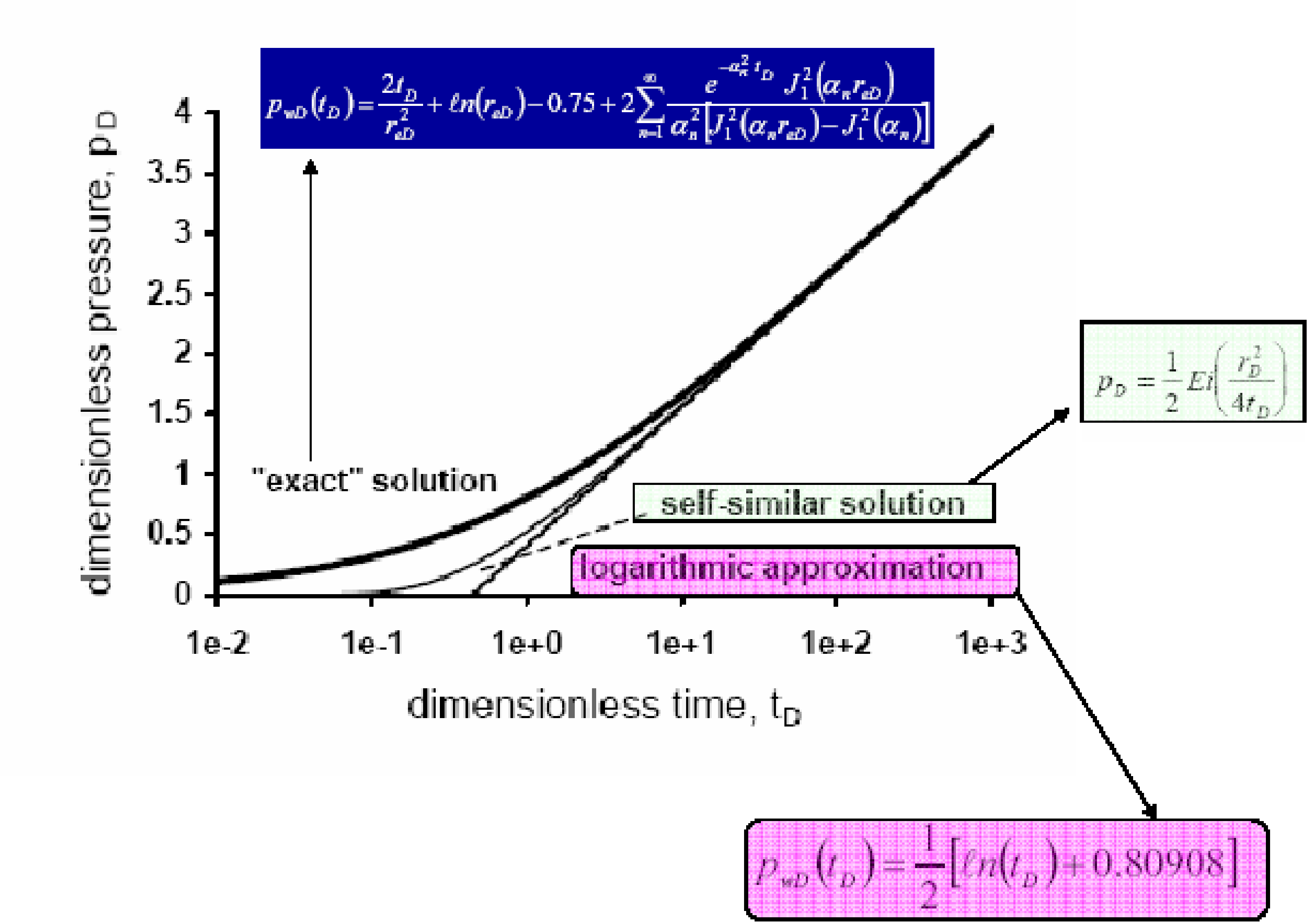
$$\frac{\partial p_{wf}}{\partial t} = \frac{0.234 qB_o}{c_t V_p}$$

Deliverability Equation & Well Productivity Index

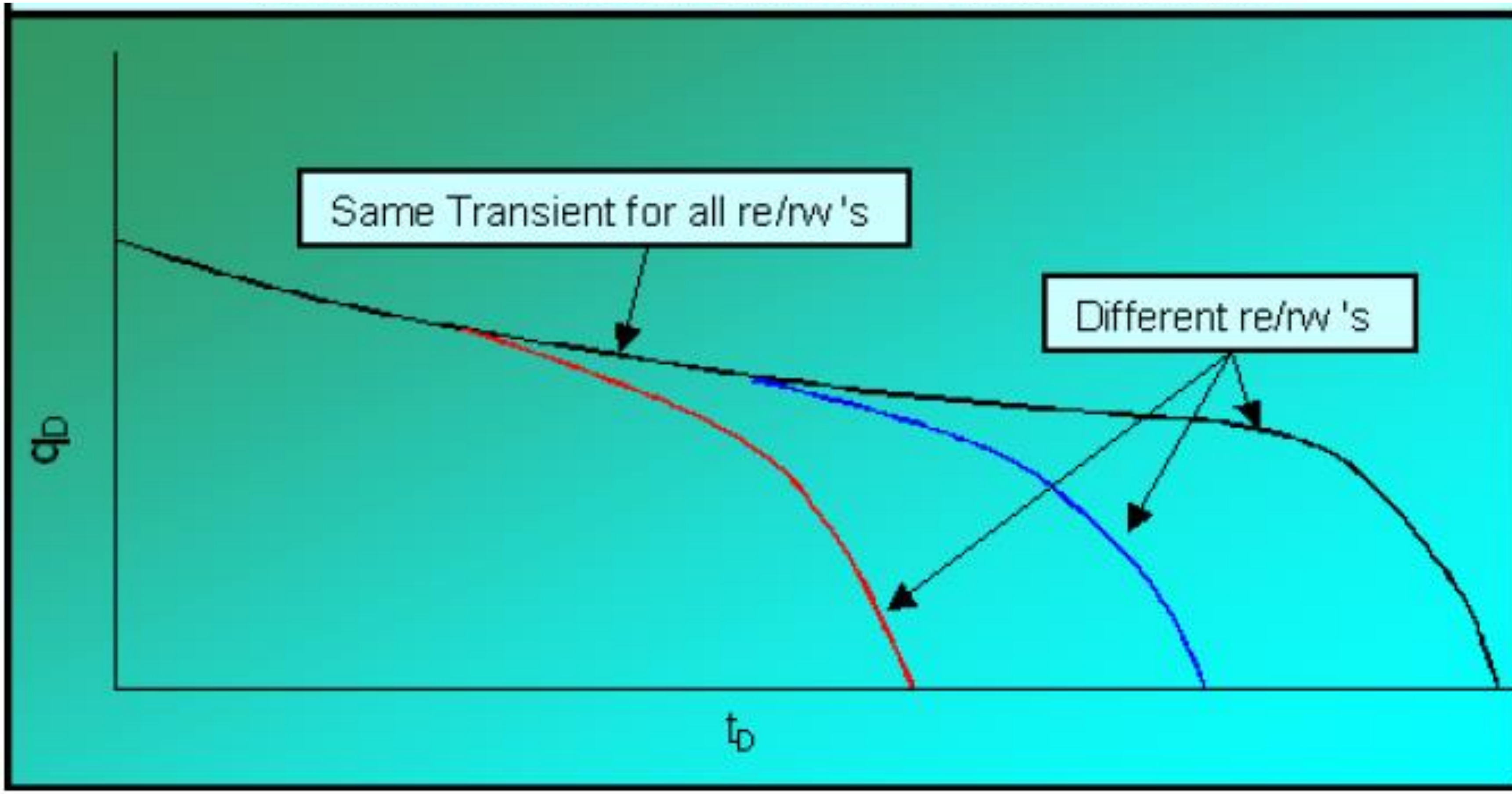
$$\bar{p} - p_{wf} = \frac{141.2 qB\mu}{kh}\left[\ell n\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

$$J = \frac{q}{\bar{p} - p_{wf}} = \frac{kh}{141.2 B\mu\left[\ell n\left(\frac{r_e}{r_w}\right) - 0.75\right]}$$

Dimensionless transient pressure response of a radial well in infinite reservoir



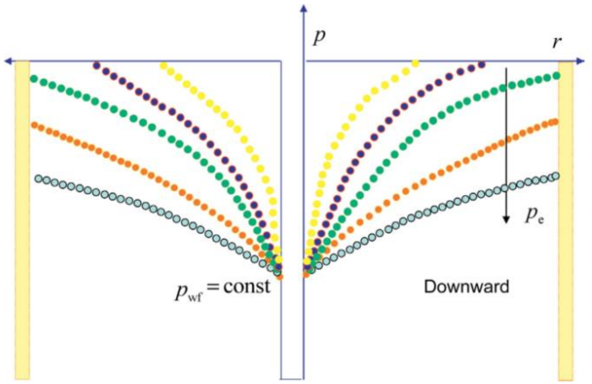
Boundary Dominated Flow



- Transient flow is independent of reservoir size, all reservoirs would follow the same curve at early time (transient flow) and would only deviate at late times, when the reservoir boundary is felt.
- The higher the reservoir size, the longer the transient flow
- The late-time behavior for all reservoir sizes is an exponential decline.

Depletion above the Bubble-point Pressure Constant Pressure Solution

$$q_D\left(t_D\right)=\frac{1}{\ln\dfrac{4A}{\gamma C_A r_w^2}}\exp\left(\frac{-4\pi t_{DA}}{\mu\ln\dfrac{4A}{\gamma C_A r_w^2}}\right)$$



$$For \qquad t_{DA}>\left(t_{pss}\right)_D$$

It may be used for reservoir limit test

$$\ln\left(q\right)=-\frac{4\pi t_{DA}}{\mu\ln\left(\dfrac{4A}{\gamma C_A r_w^2}\right)}+\ln\frac{4\pi kh\left(p_i-p_{wf}\right)}{\mu B\ln\left(\dfrac{4A}{\gamma C_A r_w^2}\right)}$$

Solutions- Laplace Domain (Sabet, 1991).

Constant rate solution	**Infinite-acting reservoir**	$\bar{p}_D(S)=\frac{K_0\left(r_D\sqrt{S}\right)}{S\sqrt{S}\,K_1\left(\sqrt{S}\right)}$
	Bounded reservoir	$\bar{p}_D(S)=\frac{\left[K_1\left(r_{De}\sqrt{S}\right)I_0\left(r_D\sqrt{S}\right)+I_1\left(r_{De}\sqrt{S}\right)K_0\left(r_D\sqrt{S}\right)\right]}{S\sqrt{S}\left[K_1\left(\sqrt{S}\right)I_1\left(r_{De}\sqrt{S}\right)-K_1\left(r_{De}\sqrt{S}\right)I_1\left(\sqrt{S}\right)\right]}$
Constant pressure solution	**Infinite-acting reservoir**	$\bar{q}_D(S)=\left(\frac{K_1\left(r_D\sqrt{S}\right)}{\sqrt{S}\,K_0\left(\sqrt{S}\right)}\right)$
	Bounded reservoir	$q_{wD}(S)=\frac{K_1\left(r_{De}\sqrt{S}\right)I_1\left(\sqrt{S}\right)-I_1\left(r_{De}\sqrt{S}\right)K_1\left(\sqrt{S}\right)}{\sqrt{S}\left[K_1\left(r_{De}\sqrt{S}\right)I_0\left(\sqrt{S}\right)+K_0\left(\sqrt{S}\right)I_1\left(r_{De}\sqrt{S}\right)\right]}$

37

Bessel Differential Equation

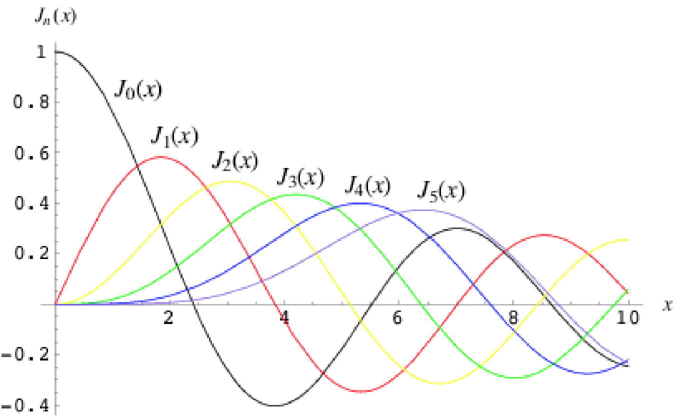
$$x^2 \frac{d^2 y}{d x^2} + x \frac{d y}{d x} + (x^2 - n^2) y = 0$$

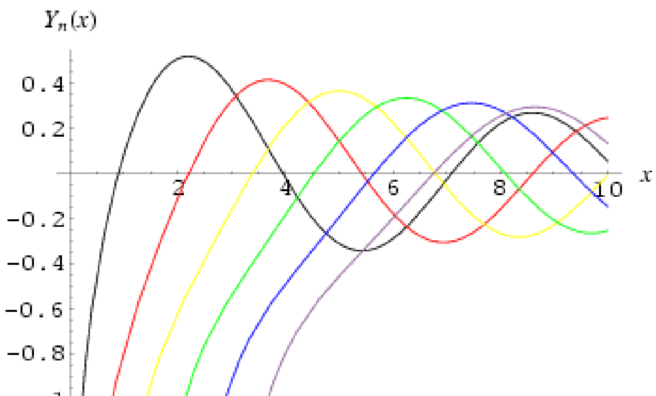
$$y(x) = c_1 J_n(x) + c_2 Y_n(x)$$

Modified Bessel Differential Equation

$$x^2 \frac{d^2 y}{d x^2} + x \frac{d y}{d x} - (x^2 + n^2) y = 0.$$

$$y(x) = c_1 I_n(x) + c_2 K_n(x)$$

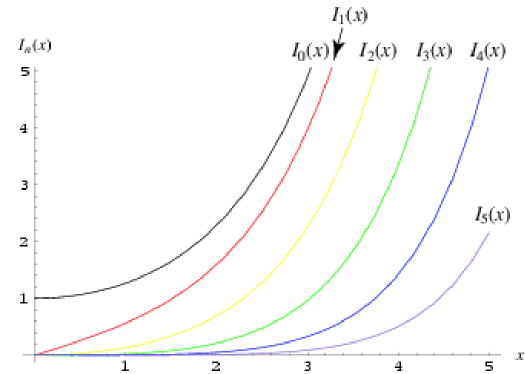


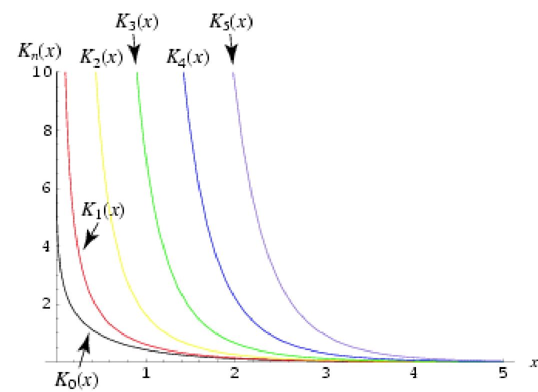


Properties of Bessel function

$$\frac{d}{dr_D} I_0\left(r_D\sqrt{S}\right) = \sqrt{S}\, I_1\left(r_D\sqrt{S}\right)$$

$$\frac{d}{dr_D} K_0\left(r_D\sqrt{S}\right) = -\sqrt{S}\, K_1\left(r_D\sqrt{S}\right)$$





Consideration of Complexibilities in the Basic Well Test Model

- **Wellbore storage**
  - Early time distorted data
- **Altered permeability in near the wellbore zone**
  - Skin factor
- **Limited flow entry**
  - Pseudo skin factor
- **Non-Darcy flow**
  - Rate-dependent skin factor
- **Multiple well**
  - Super-position in space
- **Variable rate**
  - Superposition in time
- **Compressible flow**
  - Single phase pseudo pressure
- **Two phase flow**
  - Multi-phase pseudo pressure

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]=\frac{\varphi\mu c_t}{k}\frac{\partial p}{\partial t}$$

$p = p_i$	$r_w \le r \le r_e$
$q =$ Const.	@ $r = r_w$
$q = 0$	@ $r = r_e$

Components of Well Test Models

- **Well**
  - Direction (Vertical, Horizontal)
  - Storage (Constant, Changing)
  - Completion (Damaged, Fractured and Acidized)
- **Reservoir**
  - Homogeneous
  - Heterogeneous
  - Composite
  - Multilayer
  - Dual porosity
- **Boundaries**
  - Flow boundaries (No flow, Constant pressure, infinite)
  - Geometrical boundaries (Circular, Rectangular)

Dimensionless Variables
Radial Flow With WBS And Skin

$$p_D \equiv \frac{kh\left(p_i - p\right)}{141.2qB\mu}$$

$$t_D \equiv \frac{0.0002637kt}{\phi\mu c_t r_w^2}$$

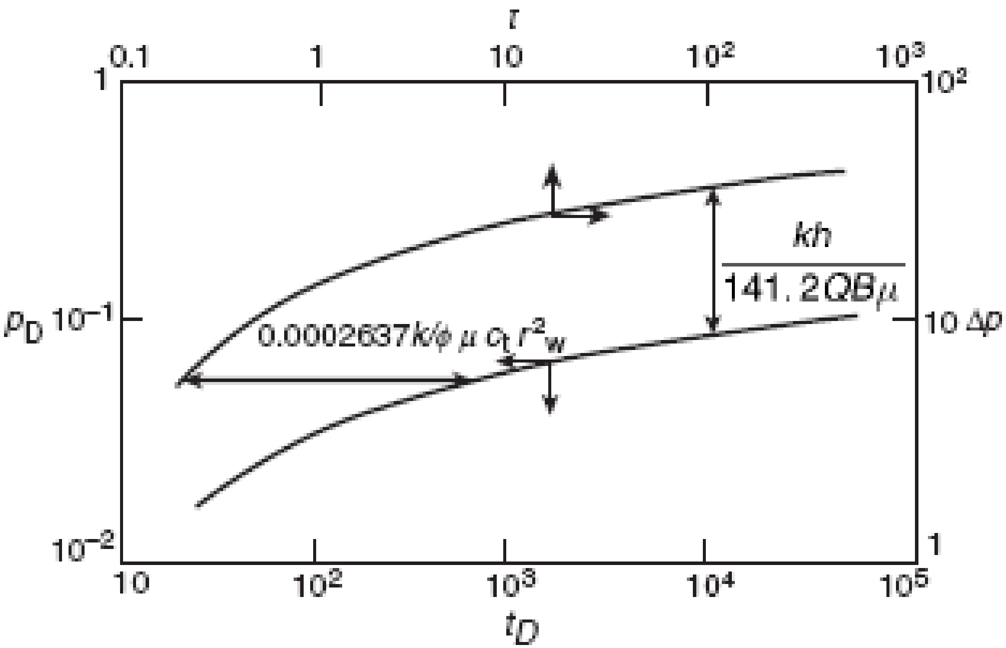
$$r_D \equiv \frac{r}{r_w}$$

$$s \equiv \frac{kh\Delta p_s}{141.2qB\mu}$$

$$C_D \equiv \frac{0.8936C}{\phi c_t h r_w^2}$$

Type Curve Matching Principle

A type curve is a graphical representation of the theoretical solutions to flow equations.



$$p_D = \left[\frac{kh}{141.2QB\mu}\right]\Delta p$$

$$\log(p_D) = \log(\Delta p) + \log\left(\frac{kh}{141.2QB\mu}\right)$$

$$t_D = \left[\frac{0.0002637k}{\phi\mu c_t r_w^2}\right]t$$

$$\log(t_D) = \log(t) + \log\left[\frac{0.0002637k}{\phi\mu c_t r_w^2}\right]$$

In other words, a dimensionless curve is different from a dimensional curve only by a constant on the log–log paper.

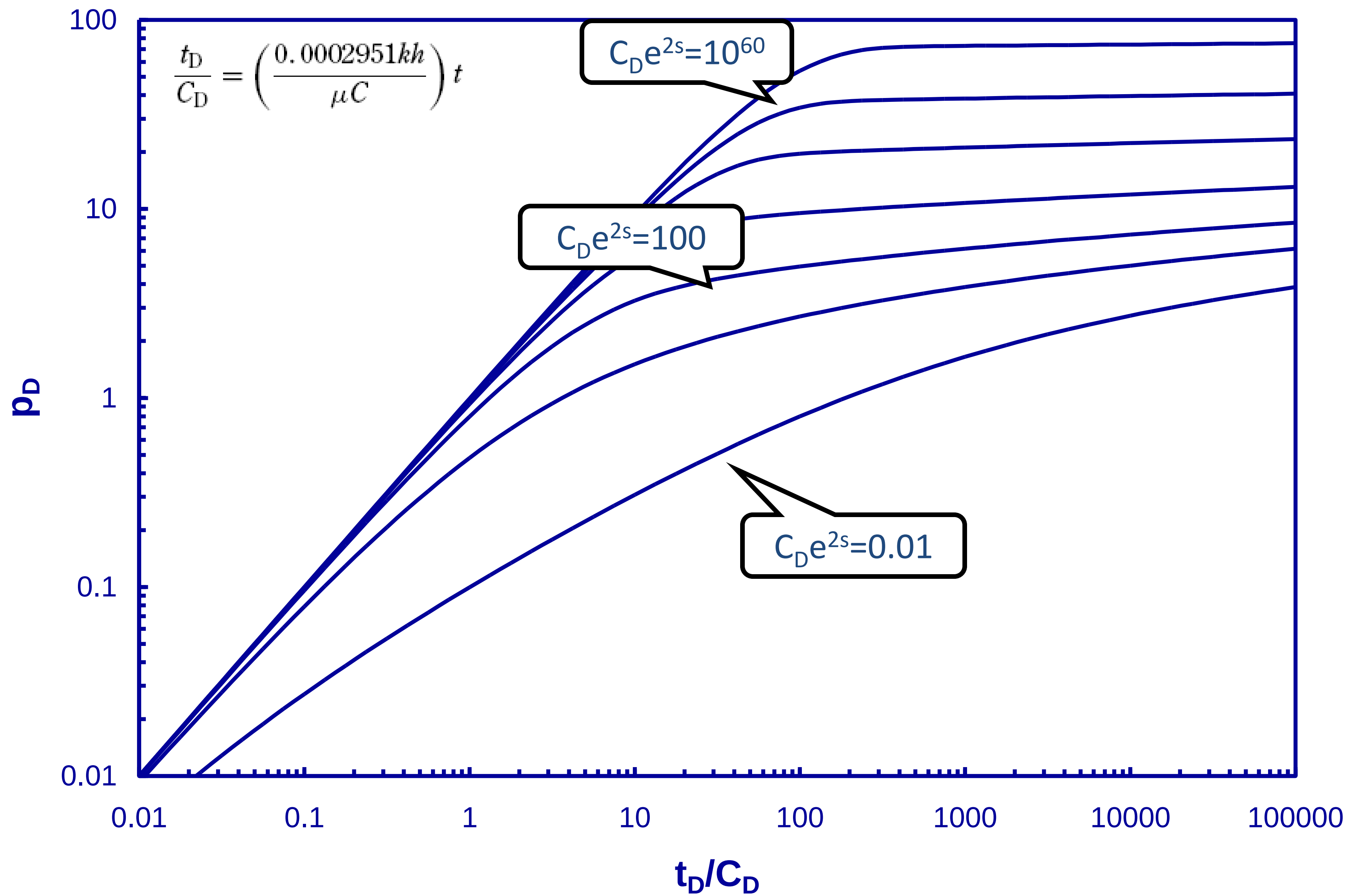
$$\phi = \frac{0.0002637k}{\mu c_t r^2[(t_D/r_D^2)/t\}_{MP}}$$

$$k = \frac{141.2QB\mu}{h}\left(\frac{p_D}{\Delta p}\right)_{MP}$$

Gringarten Type Curve

- Constant rate production
- Vertical well
- Infinite-acting homogeneous reservoir
- Single-phase, slightly compressible liquid
- Skin factor can be modeled with an apparent radius
- Constant wellbore storage coefficient

Pressure Type Curve



Derivative Analysis

- **Derivative**: the slope of the semi-log plot of pressure versus time.
- Perhaps one of the major advantages in using the pressure derivative in conjunction with pressure is the identification of the flow regime.
  - wellbore storage,
  - skin,
  - closed outer boundary,
  - vertically fractured well and others.

Derivative Analysis: Transient Radial Flow Regime

$$p_i - p_{wf}(t) = \Delta p_{wf} = \frac{162.6 q B_o \mu}{kh}\left[\log(t) + \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) - 3.23 + 0.87S\right]$$

$$\frac{d\Delta p_{wf}}{d\log t} = \frac{162.6 q B_o \mu}{kh}$$

$$\log\left(\frac{d\Delta p_{wf}}{d\log t}\right) = 0 \times \log(t) + \log\left(\frac{162.6 q B_o \mu}{kh}\right)$$

Derivative Analysis: P.S.S Radial Flow Regime

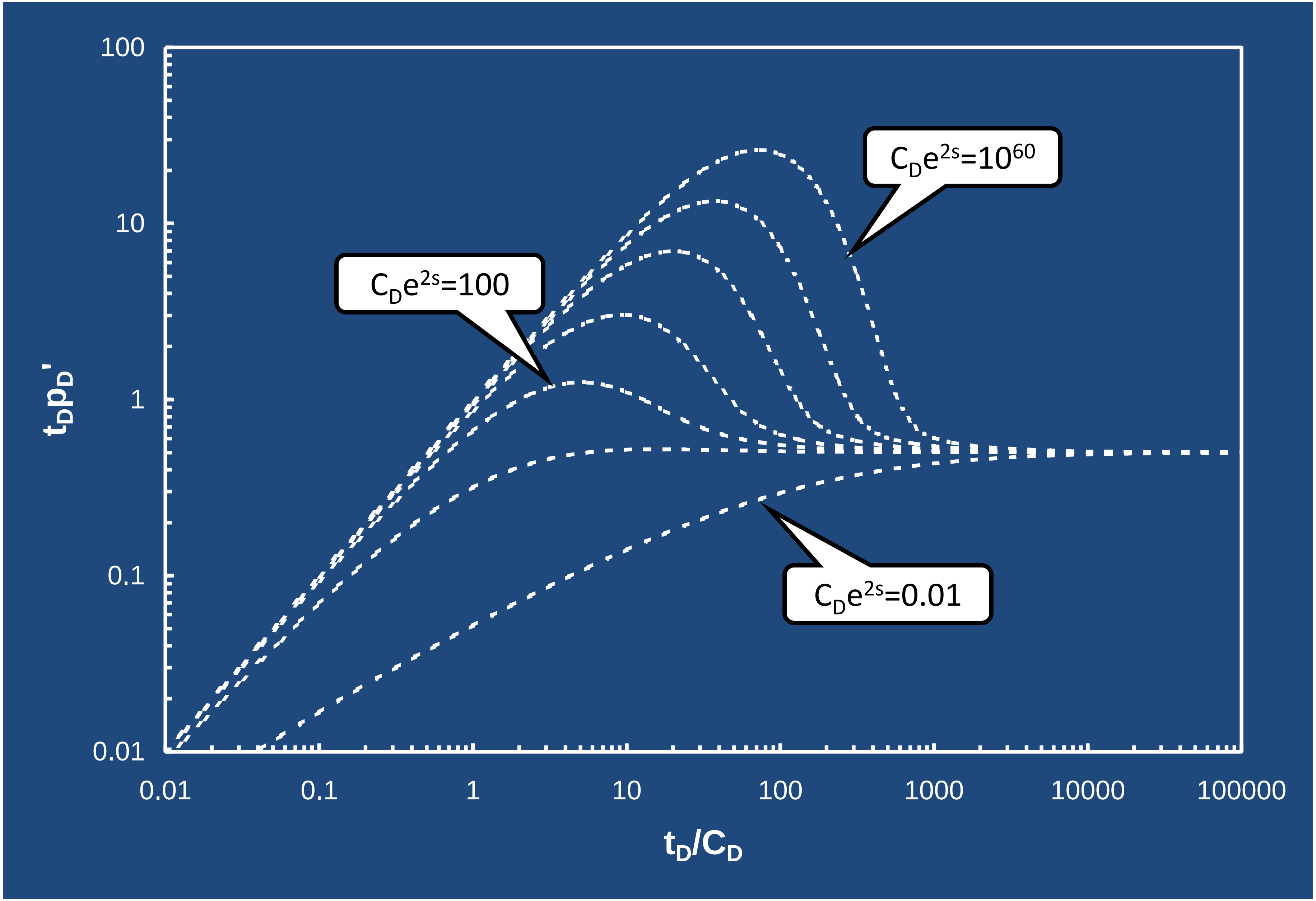
$$p_i - p_{wf} = \Delta p_{wf} = \frac{141.2qB\mu}{kh}\left[\frac{0.0005274k}{\phi\mu c_t r_e^2}t + \ell n\left(\frac{r_e}{r_w}\right) - 0.75 + S\right]$$

$$d\log t = \frac{1}{t}dt$$

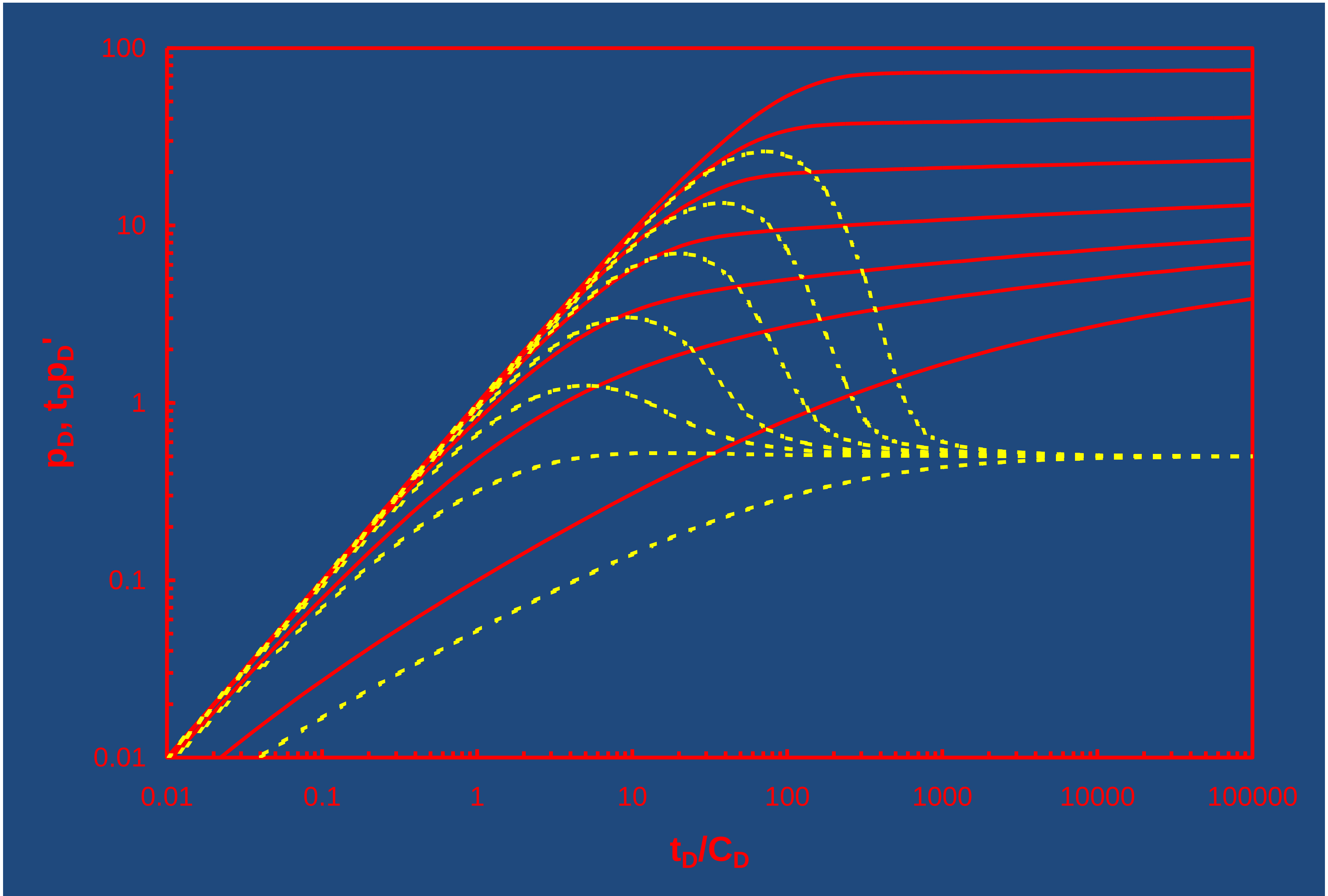
$$\frac{d\Delta p_{wf}}{d\log t} = 2.3026\frac{d\Delta p_{wf}}{dt}t = t\frac{0.1715qB_o}{\phi hc_t r_e^2}$$

$$\log\left(\frac{d\Delta p_{wf}}{d\log t}\right) = 1\times\log(t) + \log\left(\frac{0.1715qB_o}{\phi hc_t r_e^2}\right)$$

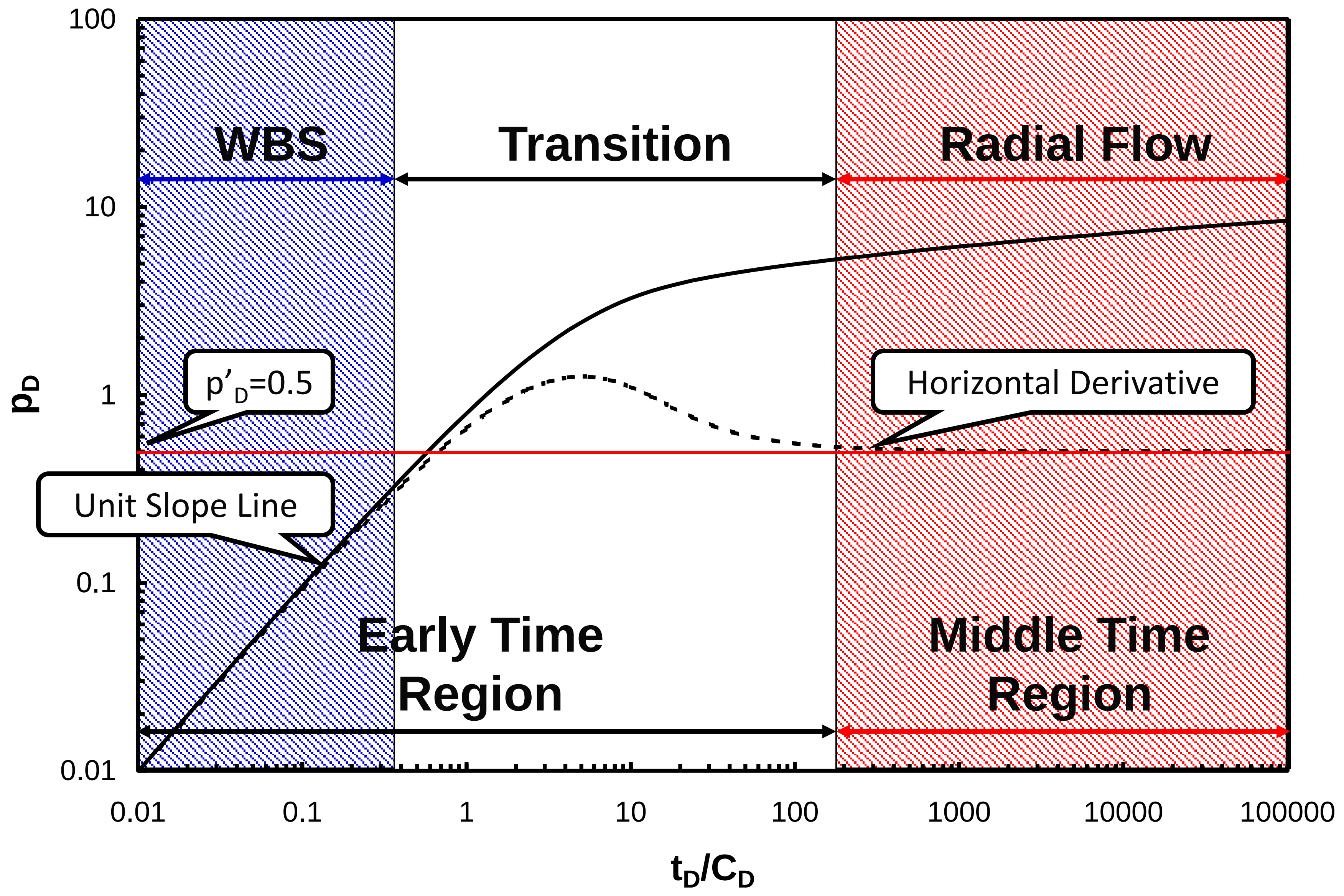
Derivative Type Curve

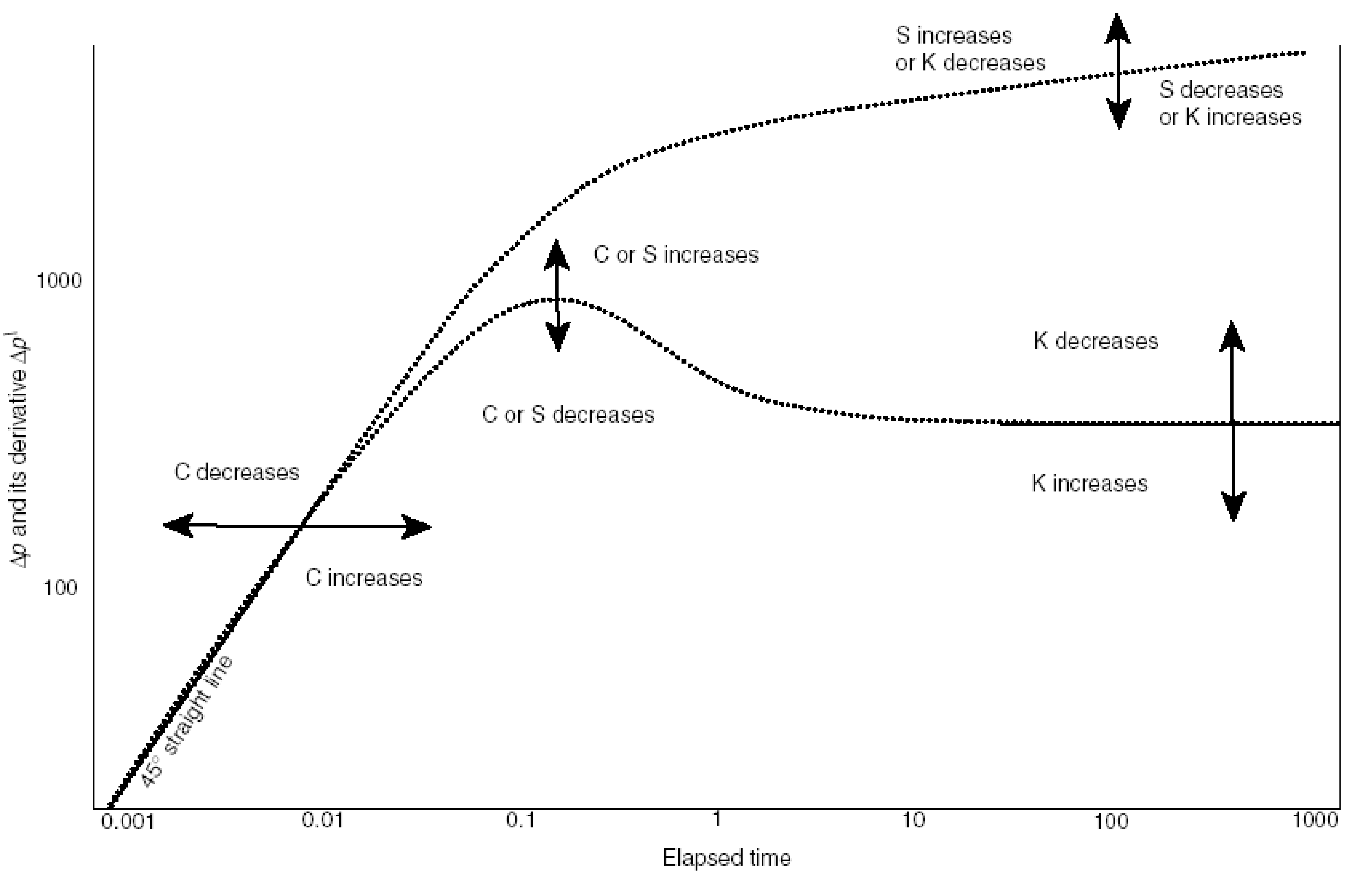


Pressure And Derivative Type Curves

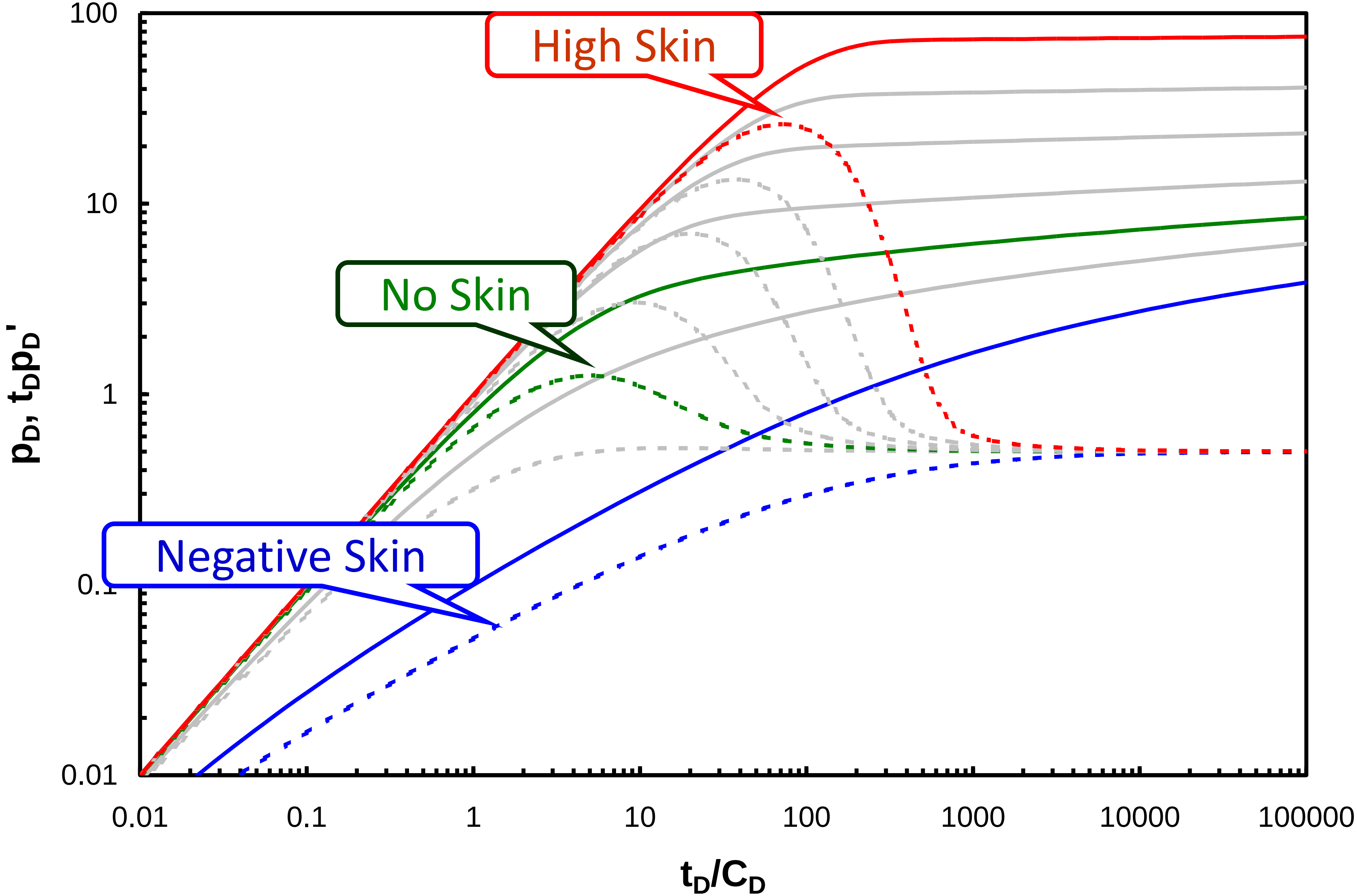


Time Regions On The Type Curve




S increases
or K decreases
S decreases
or K increases
C or S increases
1000
K decreases
C or S decreases
Δp and its derivative Δp'
C decreases
K increases
C increases
100
45° straight line
0.001
0.01
0.1
1
10
100
1000
Elapsed time

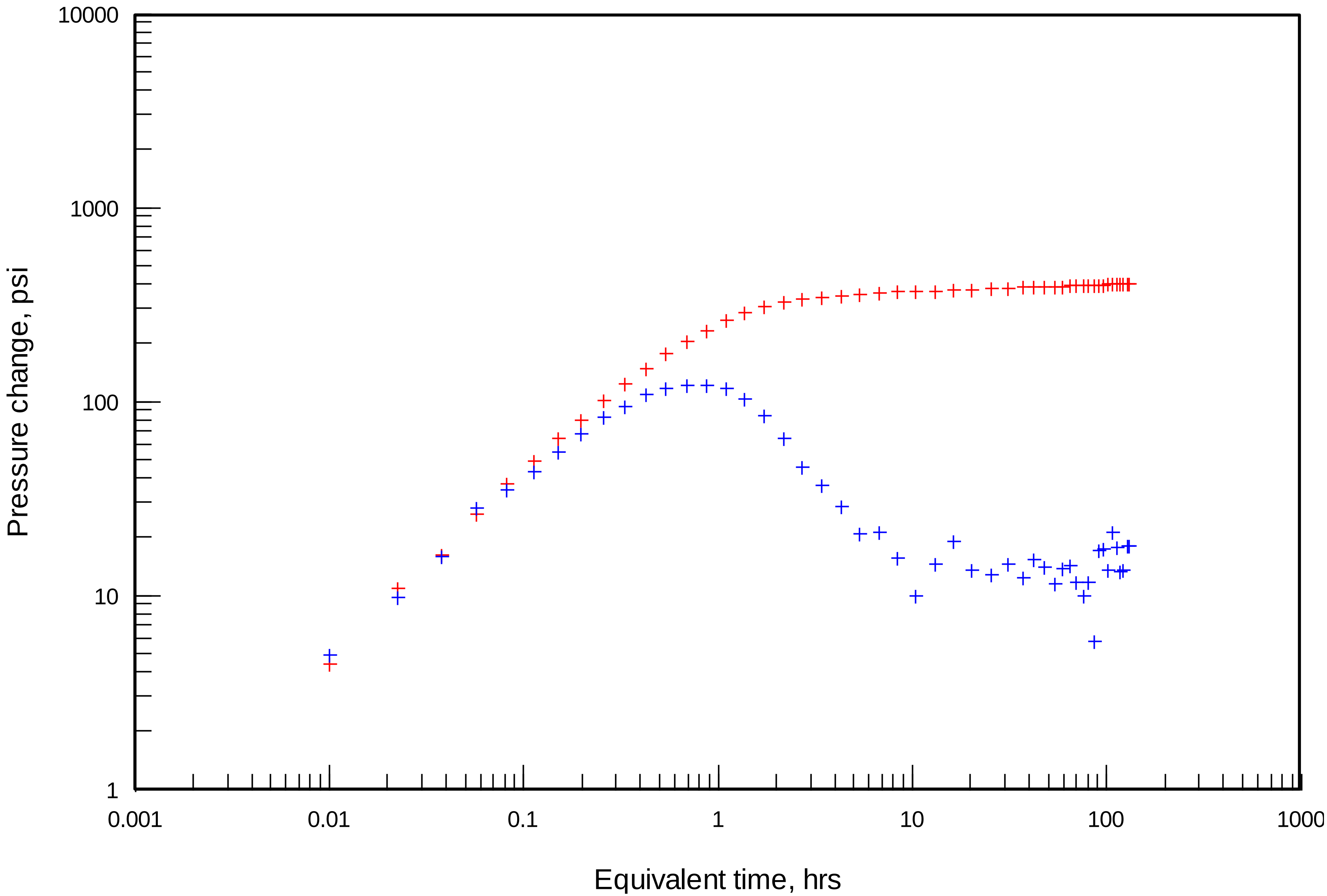
Estimating Skin Factor

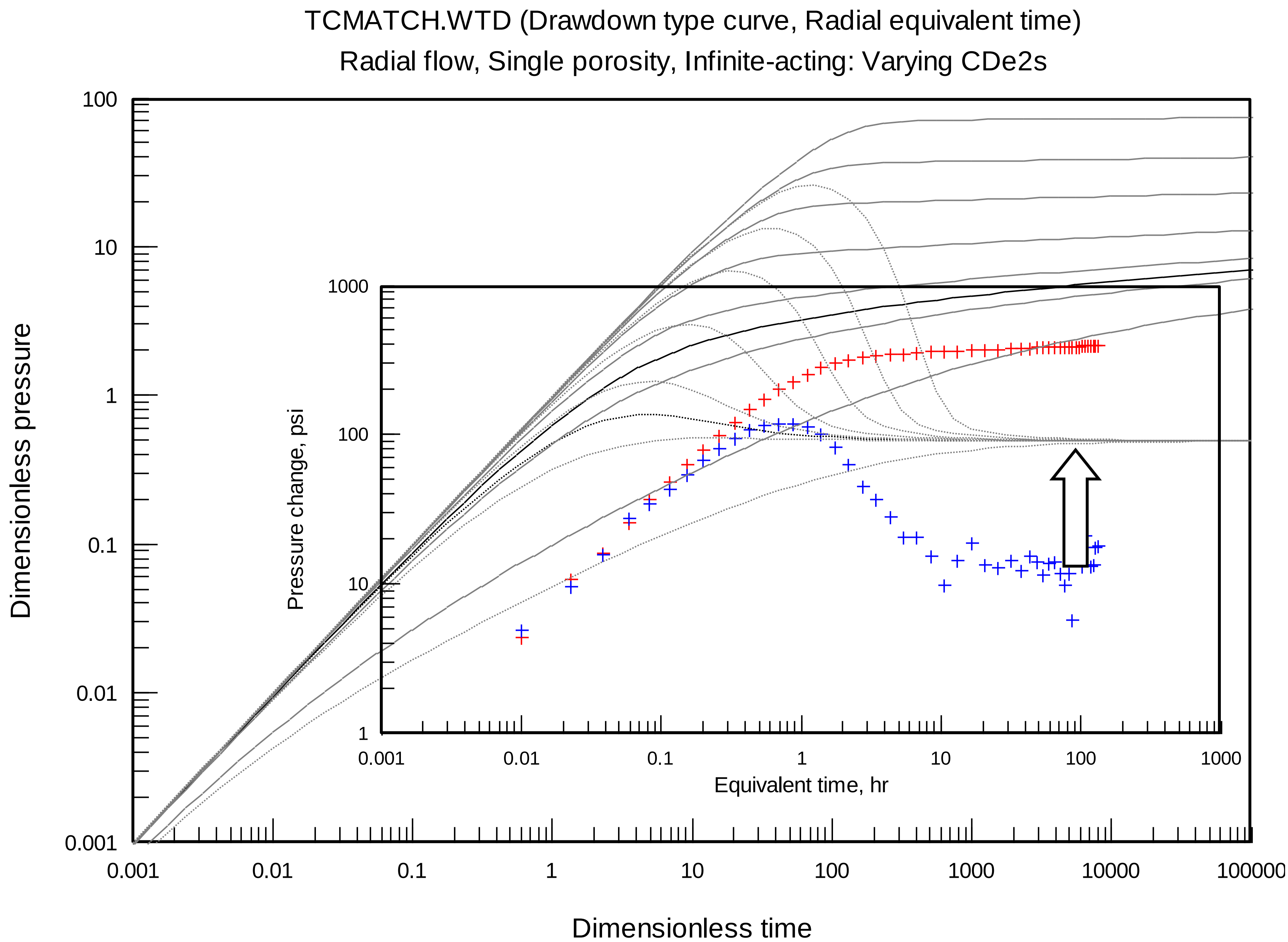


Type Curve Matching

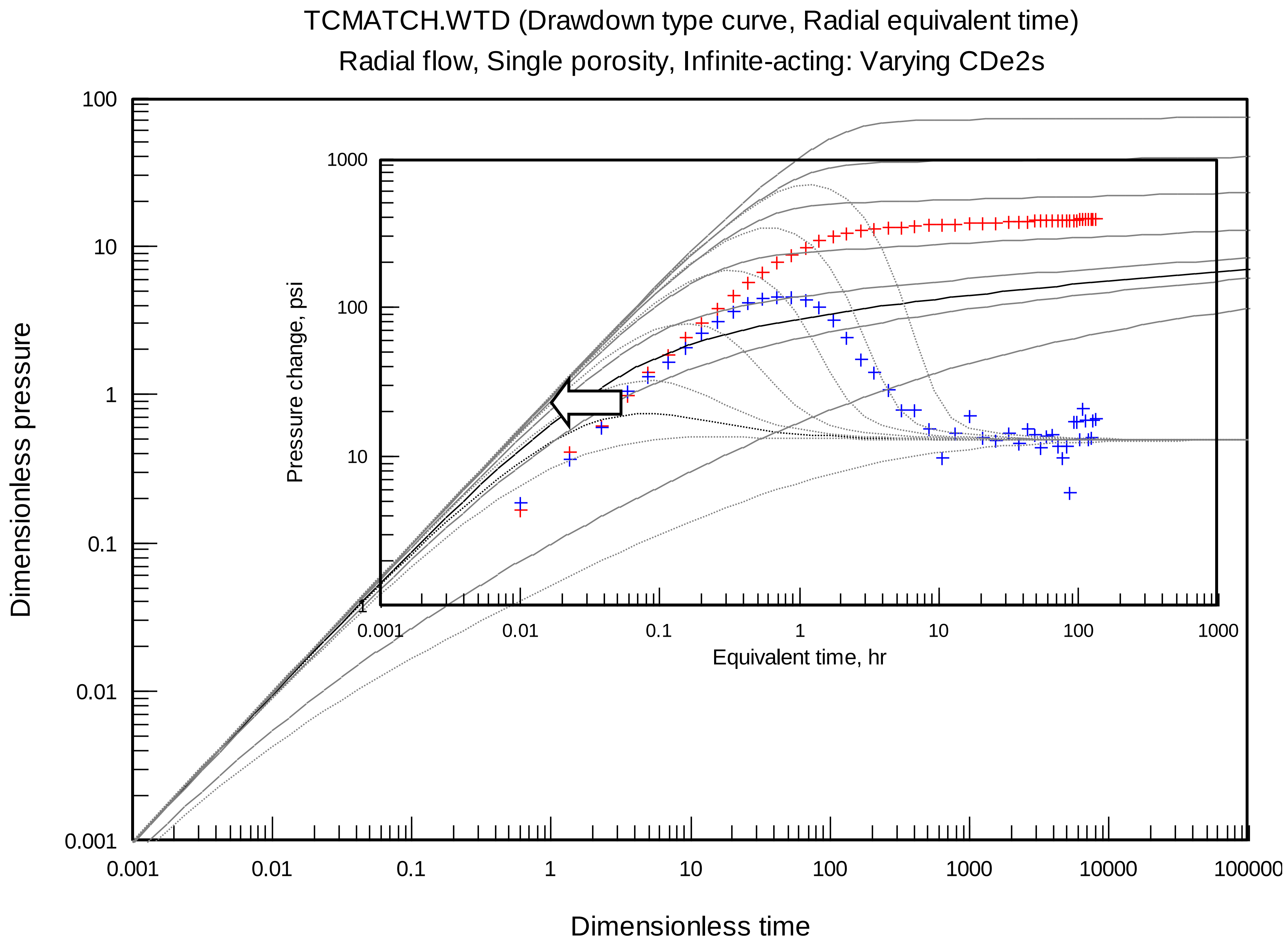
- Plot field data on log-log scale
- Align horizontal part of field data and type curve derivative
- Align unit slope part of field data and type curve
- Select value of C_De^{2s} that best matches field data

TCMATCH.WTD (Field Data)





TCMATCH.WTD (Drawdown type curve, Radial equivalent time)
Radial flow, Single porosity, Infinite-acting: Varying CDe2s
Dimensionless pressure
Dimensionless time
100
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10
100
1000
10000
100000
Pressure change, psi
Equivalent time, hr
1000
100
10
1
0.001
0.01
0.1
1
10
100
1000



TCMATCH.WTD (Drawdown type curve, Radial equivalent time)
Radial flow, Single porosity, Infinite-acting: Varying CDe2s
Dimensionless pressure
Dimensionless time
100
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10
100
1000
10000
100000
Pressure change, psi
Equivalent time, hr
1000
100
10
1
0.001
0.01
0.1
1
10
100
1000

Interpreting Type Curve Match

- Calculate k from the pressure match point ratio $\Delta p/p_D$
- Calculate C_D from the time match point ratio t_{eq}/t_D
- Calculate s from the matching stem value C_De^{2s}

$$k = \frac{141.2QB\mu}{h}\left(\frac{p_D}{\Delta p}\right)_{MP}$$

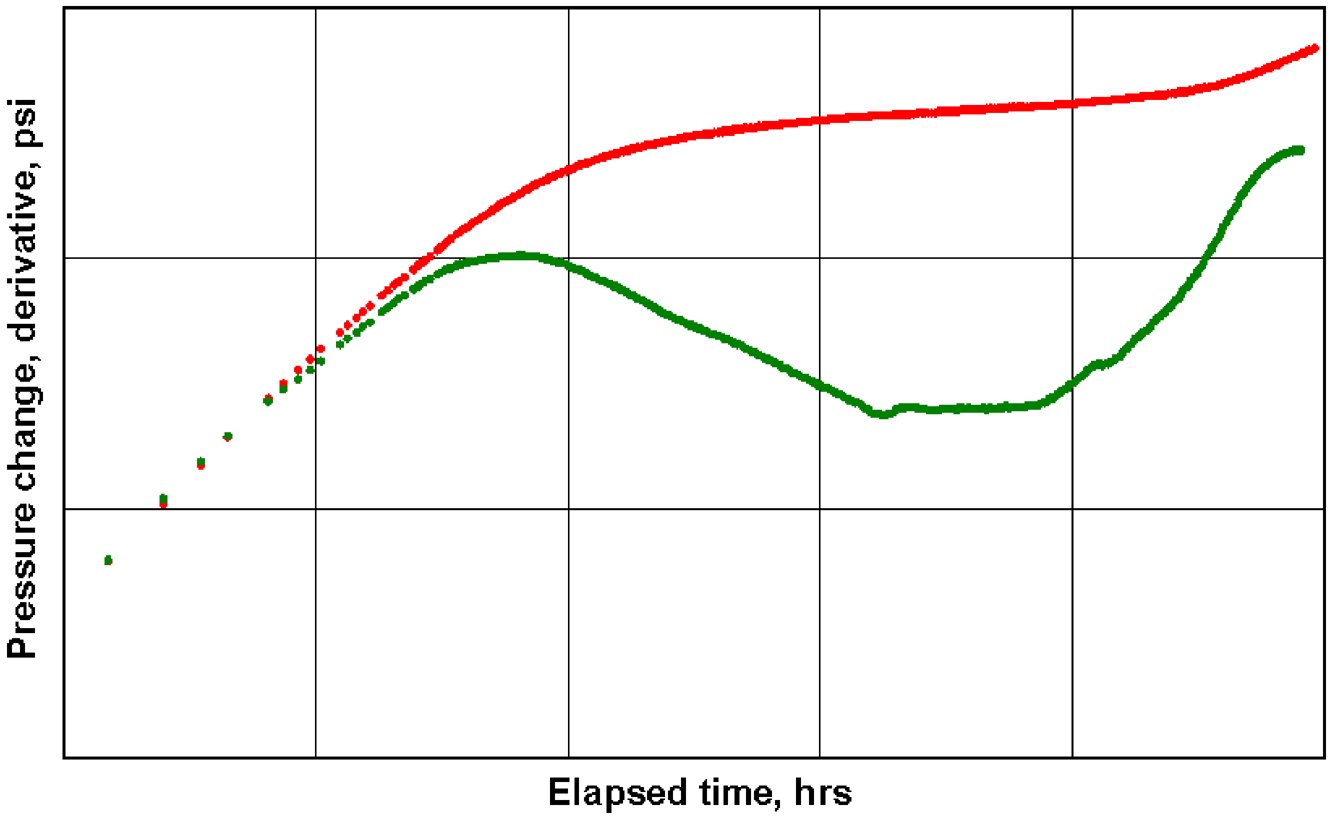
$$C = \frac{0.0002951kh}{\mu\left(\frac{t_D/C_D}{t}\right)_{MP}}$$

$$C_D = \left[\frac{0.8936}{\phi h c_t r_w^2}\right] C$$

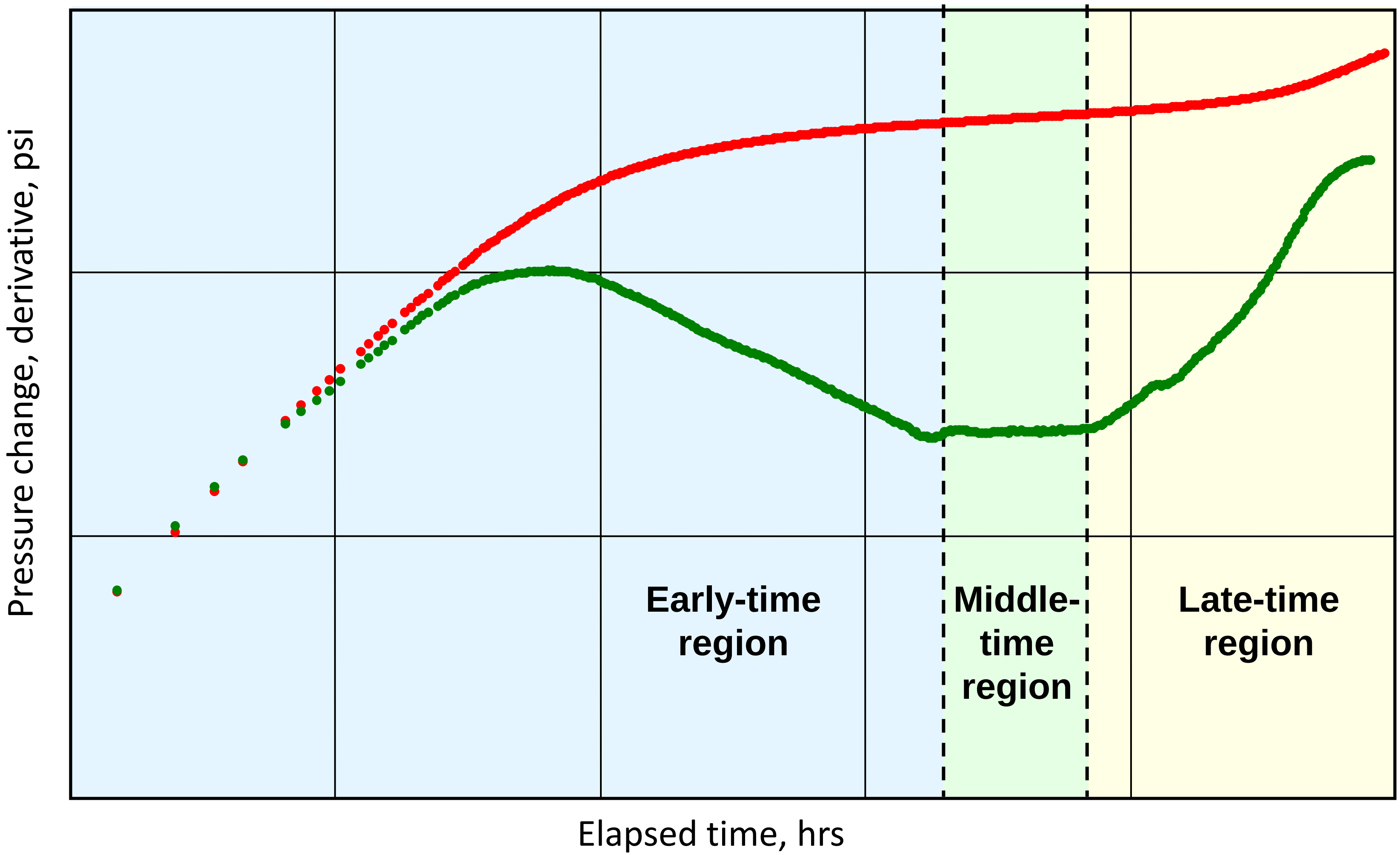
$$s = \frac{1}{2}\ln\left[\frac{(C_D e^{2s})_{MP}}{C_D}\right]$$

The Diagnostic Plot

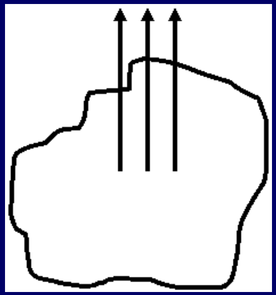
1. Identify time regions.
2. Identify flow regimes.
3. List factors that affect pressure response in early time.
4. List boundaries that affect pressure response in late time.



Time Regions on the Diagnostic Plot



Volumetric Behavior



Wellbore Storage

$$\Delta p = \frac{qBt}{24C}$$

Pseudosteady-State Flow

$$p_i - p_{wf} = \frac{0.0744qBt}{\phi c_t h r_e^2} + \frac{141.2qB\mu}{kh}\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s\right]$$

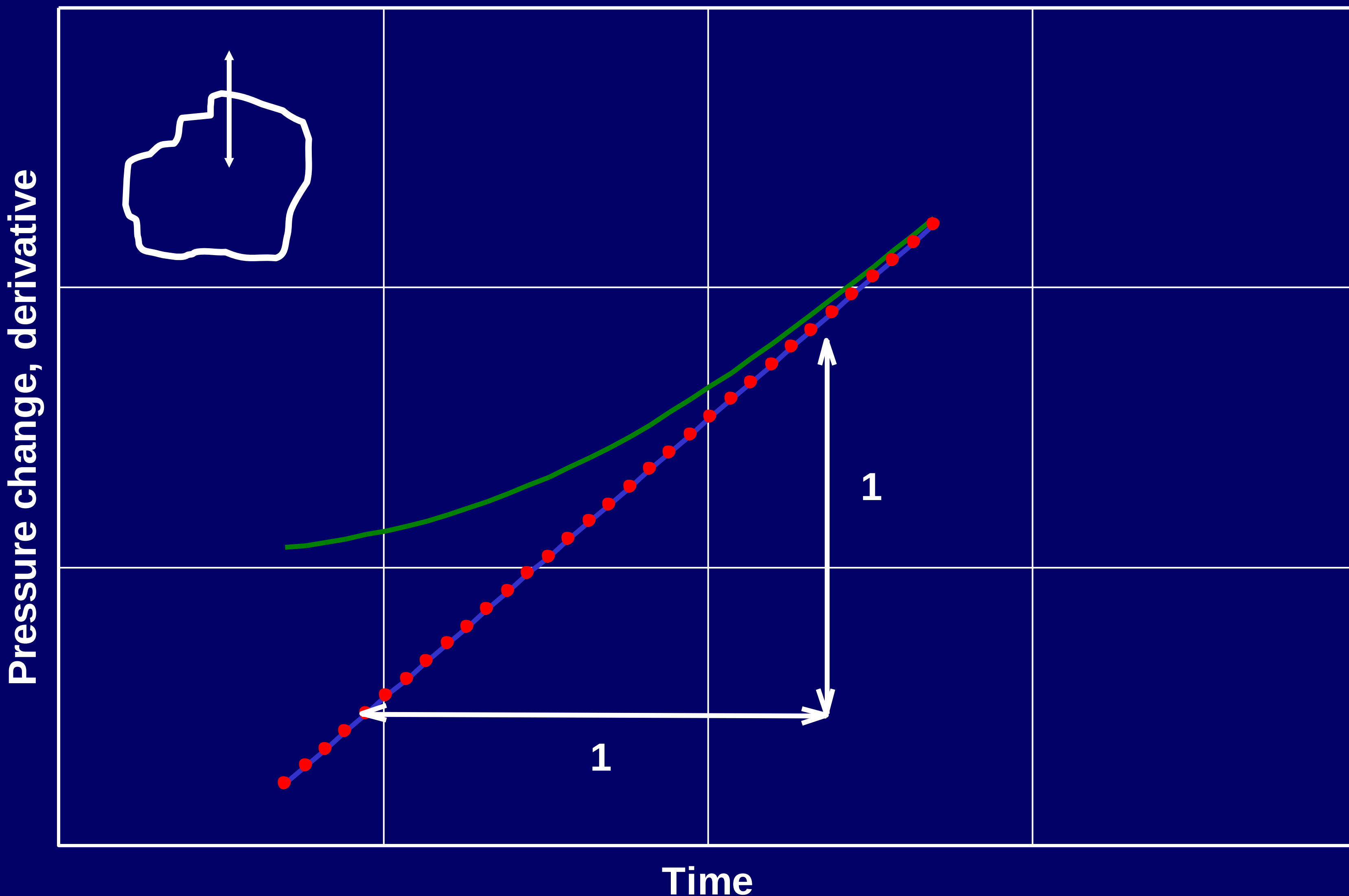
General Form

$$\Delta p = m_V t + b_V$$

Derivative

$$t\frac{\partial \Delta p}{\partial t} = t\frac{\partial\left(m_V t + b_V\right)}{\partial t} = m_V t$$

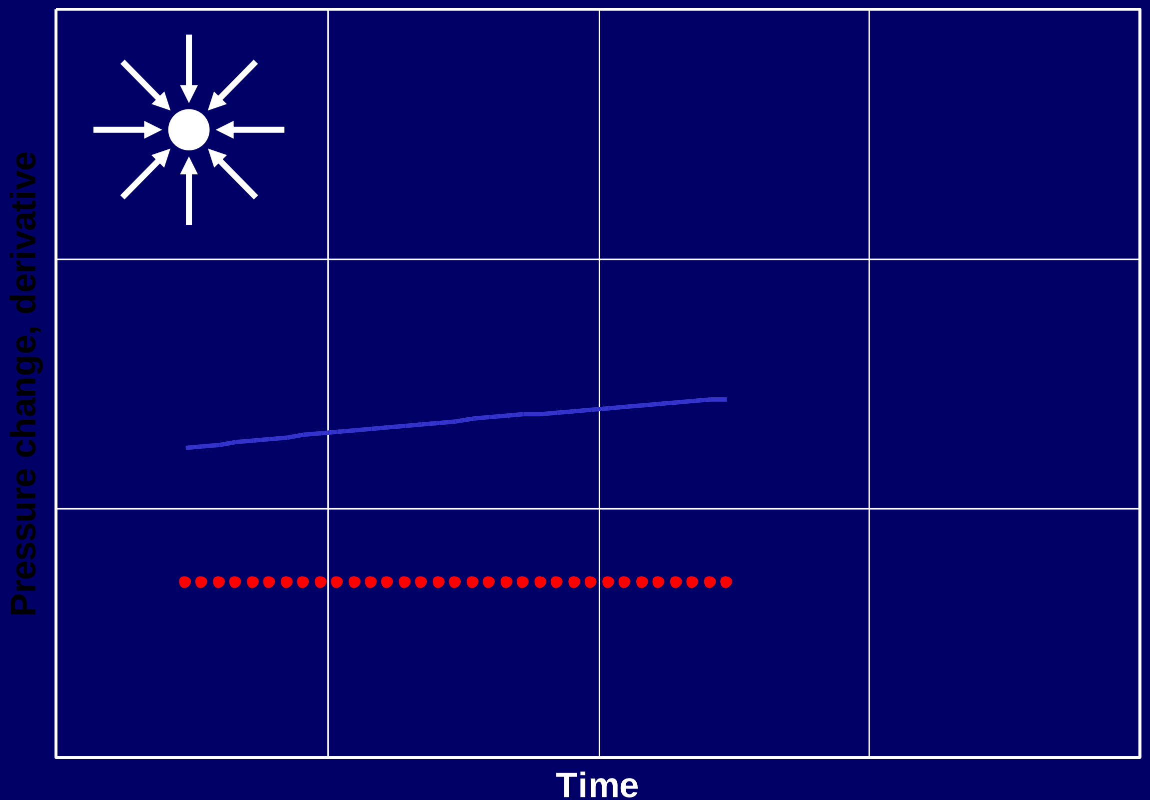
Volumetric Behavior



Radial Flow

General Form $$\Delta p = m\log(t) + b$$

Derivative $$t\frac{\partial \Delta p}{\partial t} = t\frac{\partial\left(m\log(t)+b\right)}{\partial t} = \frac{m}{2.303}$$



Linear Flow

Hydraulic Fracture

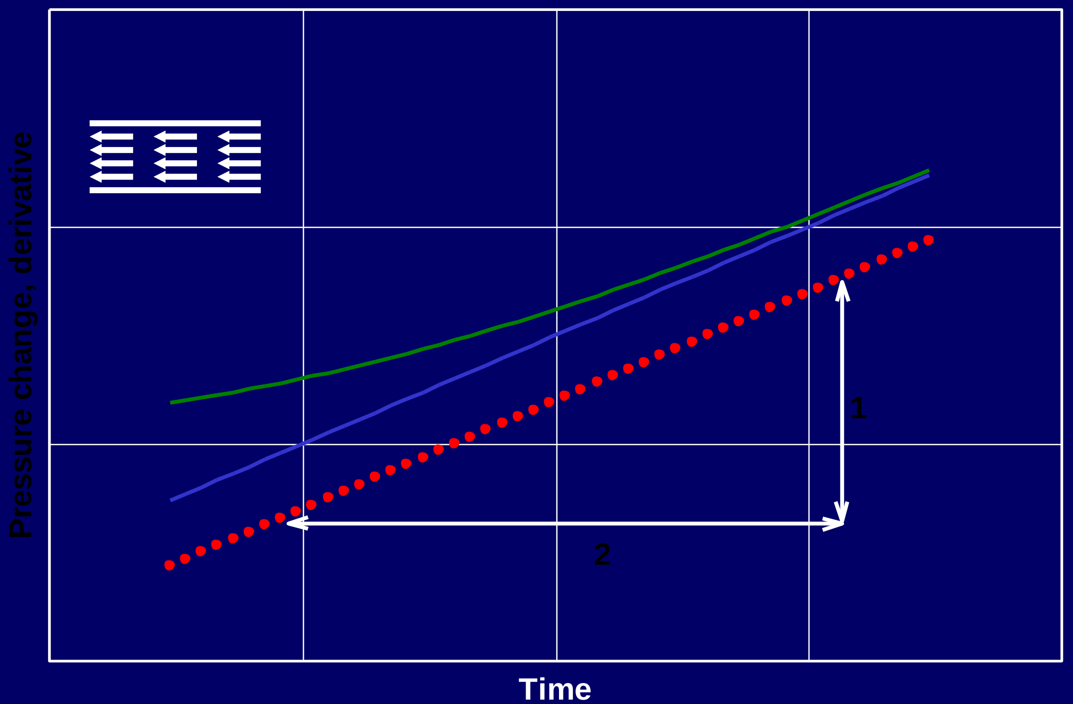
$$\Delta p = \frac{4.064 qB\mu}{khL_f}\left(\frac{kt}{\phi\mu c_t}\right)^{1/2}$$

General Form

$$\Delta p = m_L t^{1/2} + b_L$$

Derivative

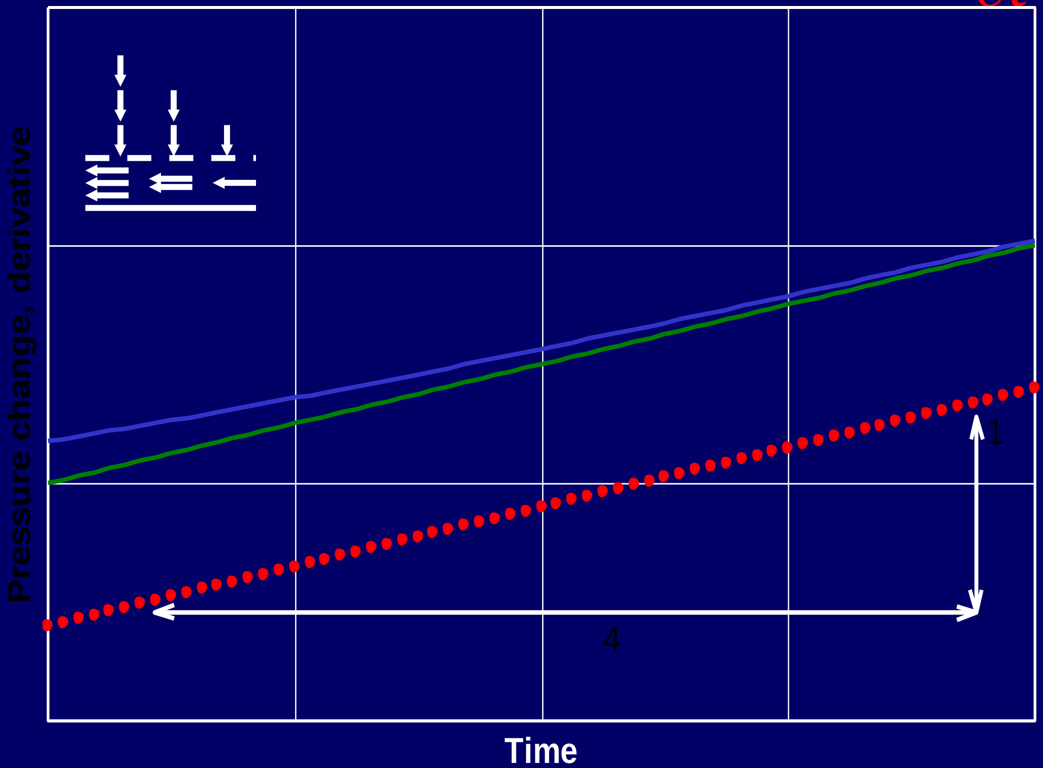
$$t\frac{\partial \Delta p}{\partial t} = t\frac{\partial\left(m_L t^{1/2} + b_L\right)}{\partial t} = \frac{1}{2} m_L t^{1/2}$$



Bilinear Flow

Hydraulic Fracture

$$\Delta p = \frac{44.1 q B \mu}{h} \left(\frac{1}{w k_f} \right)^{1/2} \left(\frac{t}{\phi \mu c_t k} \right)^{1/4}$$



General Form

$$\Delta p = m_B t^{1/4} + b_B$$

Derivative

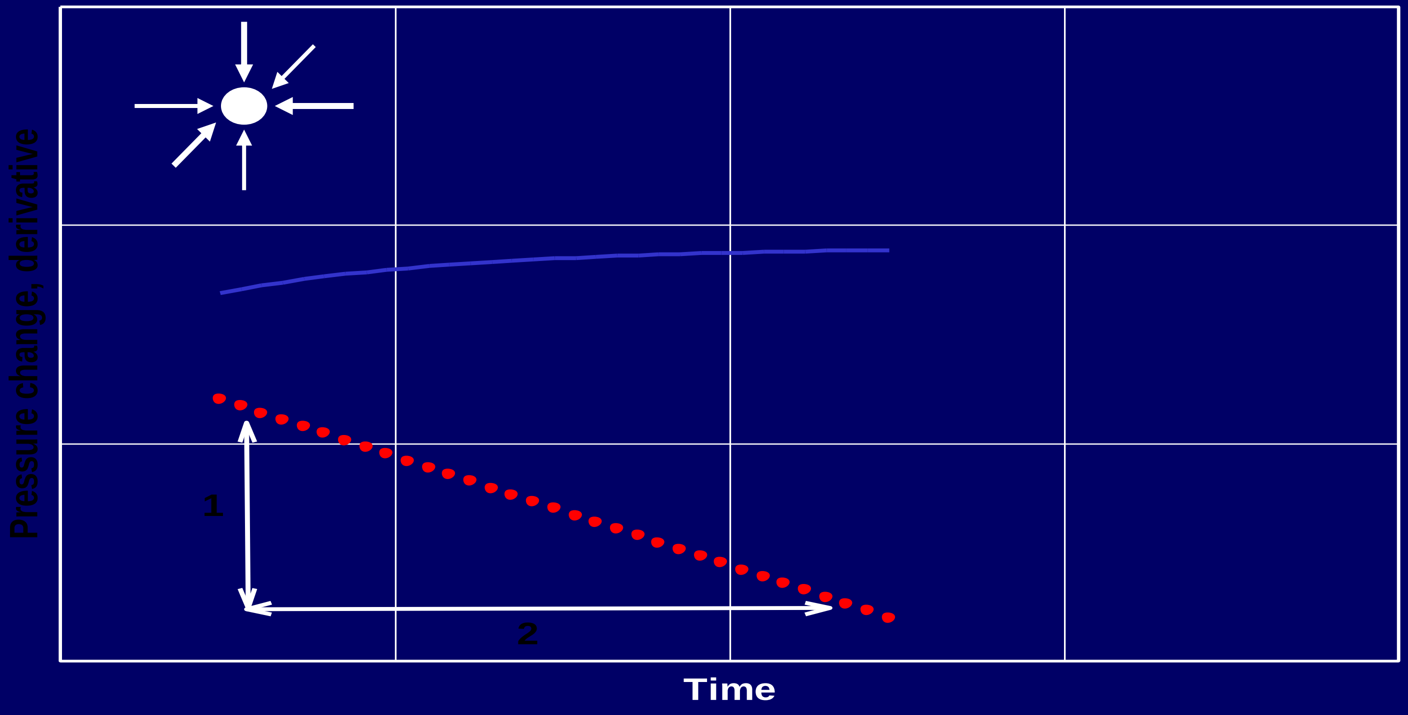
$$t \frac{\partial \Delta p}{\partial t} = t \frac{\partial \left(m_B t^{1/4} + b_B \right)}{\partial t} = \frac{1}{4} m_B t^{1/4}$$

Spherical Flow

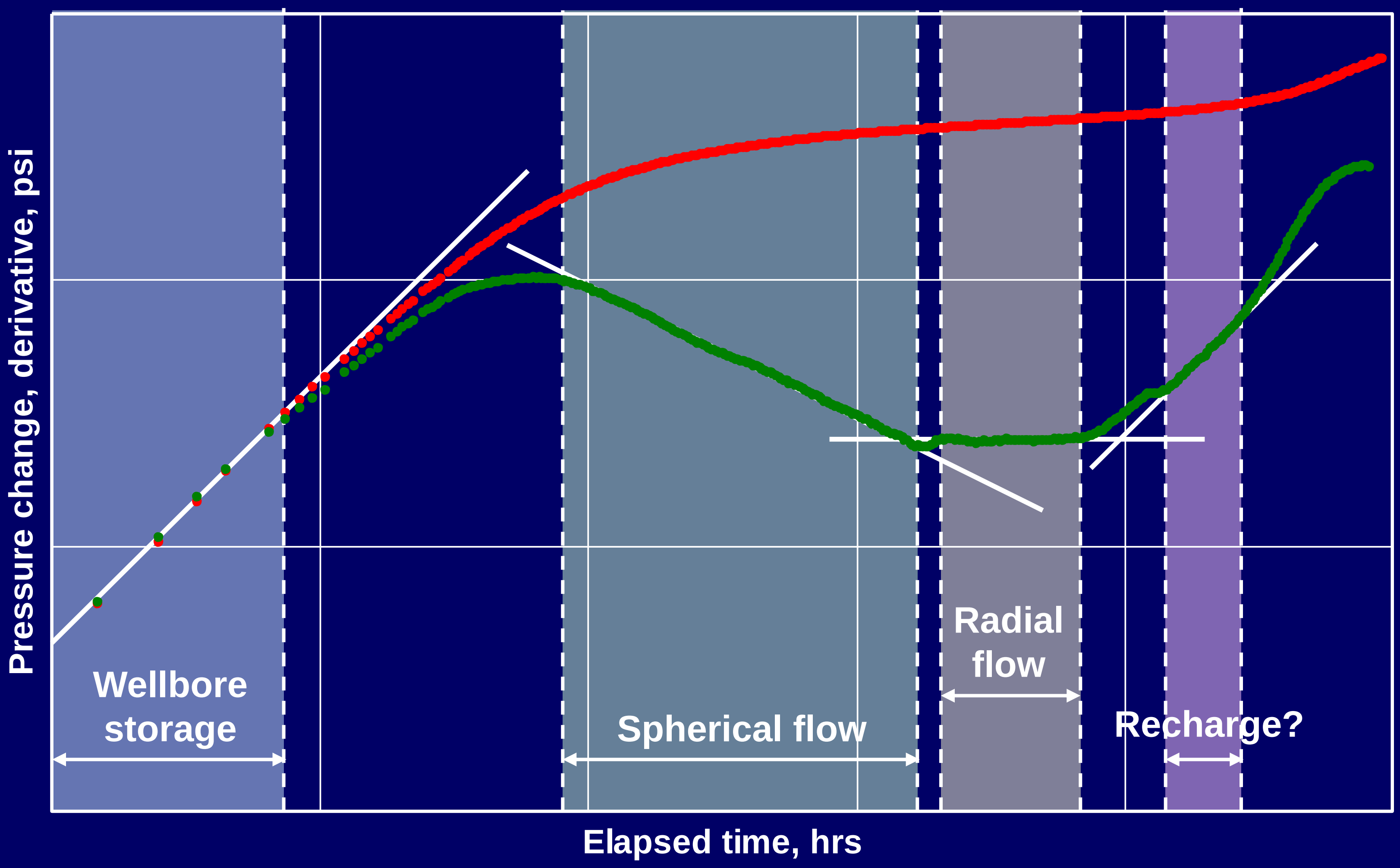
$$p_i - p_{wf} = \frac{q\mu}{4\pi k r_p}\left(1 - \sqrt{\frac{\phi\mu c_t r_p^2}{kt}}\right)$$

General Form $$\Delta p = b_S - m_S t^{-1/2}$$

Derivative $$t\frac{\partial \Delta p}{\partial t} = t\frac{\partial\left(b_s - m_S t^{-1/2}\right)}{\partial t} = \frac{1}{2} m_S t^{-1/2}$$



Flow Regimes on the Diagnostic Plot



Challenges To Deal with In Gas Reservoirs

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k\rho}{\mu} r \frac{\partial p}{\partial r}\right) = \phi c \rho \frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p}{\partial r^2} + c\left(\frac{\partial p}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\mu c \phi}{k}\frac{\partial p}{\partial t}$$

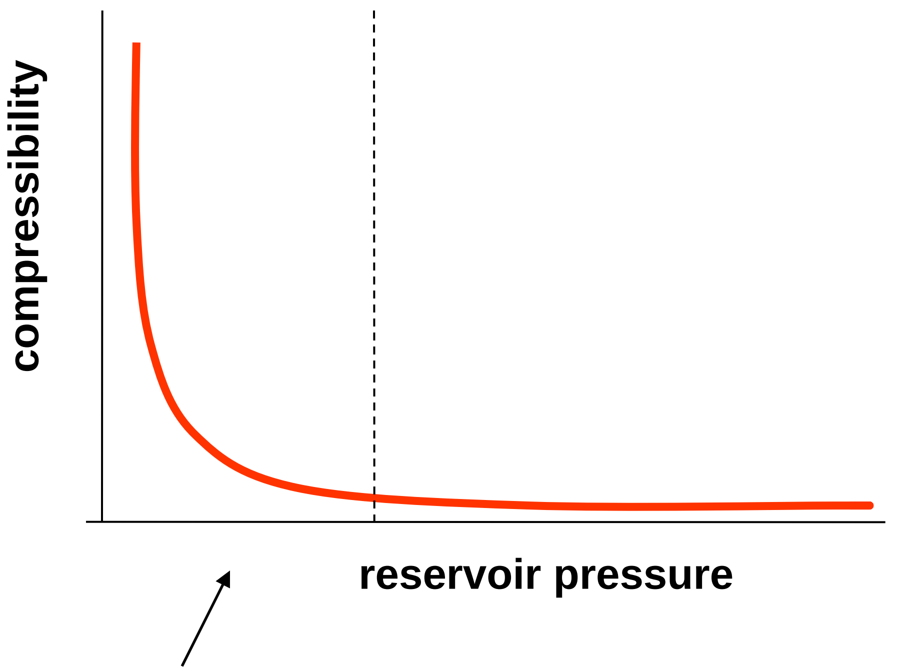
Assumptions are usually made for liquid flow:

- Viscosity is independent of pressure.
- The pressure gradient is small and therefore is negligible.
- The liquid compressibility is small and constant, so that the product.

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\mu c \phi}{k}\frac{\partial p}{\partial t}$$

Challenges To Deal with In Gas Reservoirs

1. Gas has a non-linear inflow relationship flowing wellbore pressure is not proportional to well flow rate
2. Gas properties significantly changes with pressure



For gas wells, compressibility (and viscosity) can not be considered constants at low reservoir pressure

$$\frac{\partial}{\partial t}\left(\frac{p}{Z}\right)=\frac{K}{\varphi}\nabla\cdot\left(\frac{p}{\mu Z}\nabla p\right)$$

$$m(p)=\int_{p_0}^{p}\frac{2p}{\mu Z}dp$$

$$\nabla m=\frac{\partial m}{\partial p}\nabla p=\frac{2p}{\mu Z}\nabla p$$

$$\frac{\partial m}{\partial t}=\frac{2p}{\mu Z}\frac{\partial p}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{Z}\right)=\mu C_{\mathrm{t}}\frac{p}{\mu Z}\frac{\partial p}{\partial t}$$

$$\nabla^2 m=\frac{\varphi\mu C_{\mathrm{t}}}{K}\frac{\partial m}{\partial t}$$

$$t_{\mathrm{a}}=\int_0^t\frac{\mu_{\mathrm{i}}C_{\mathrm{ti}}}{\mu C_{\mathrm{t}}}dt$$

$$\eta=\frac{K}{\varphi\mu_{\mathrm{i}}C_{\mathrm{ti}}}$$

$$\nabla^2 m=\frac{1}{\eta}\frac{\partial m}{\partial t_{\mathrm{a}}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m}{\partial r}\right)=\frac{1}{\eta}\frac{\partial m}{\partial t_{\mathrm{a}}}$$

Challenges To Deal with In Gas Reservoirs

Assuming that the gas reservoir is homogeneous and the flow follows Darcy's law, the flow equation is

$$\boxed{\frac{\partial}{\partial t}\left(\frac{p}{Z}\right)=\frac{K}{\varphi}\nabla\cdot\left(\frac{p}{\mu Z}\nabla p\right)}$$

$$m(p)=\int_{p_0}^{p}\frac{2p}{\mu Z}dp$$

Al-Hussainy and Ramey (1966)
Pseudo-pressure Transformation
(1) Variable compressibility factor
(2) Variable viscosity

$$\boxed{\nabla^2 m=\frac{\varphi\mu C_\mathrm{t}}{K}\frac{\partial m}{\partial t}}$$

$$t_\mathrm{a}=\int_0^t\frac{\mu_\mathrm{i}C_\mathrm{ti}}{\mu C_\mathrm{t}}dt$$

Fraim and Wattenbarger (1987)
Pseudo-time Transformation
(1) Variable compressibility
(2) Variable viscosity

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m}{\partial r}\right)=\frac{1}{\eta}\frac{\partial m}{\partial t_\mathrm{a}}}$$

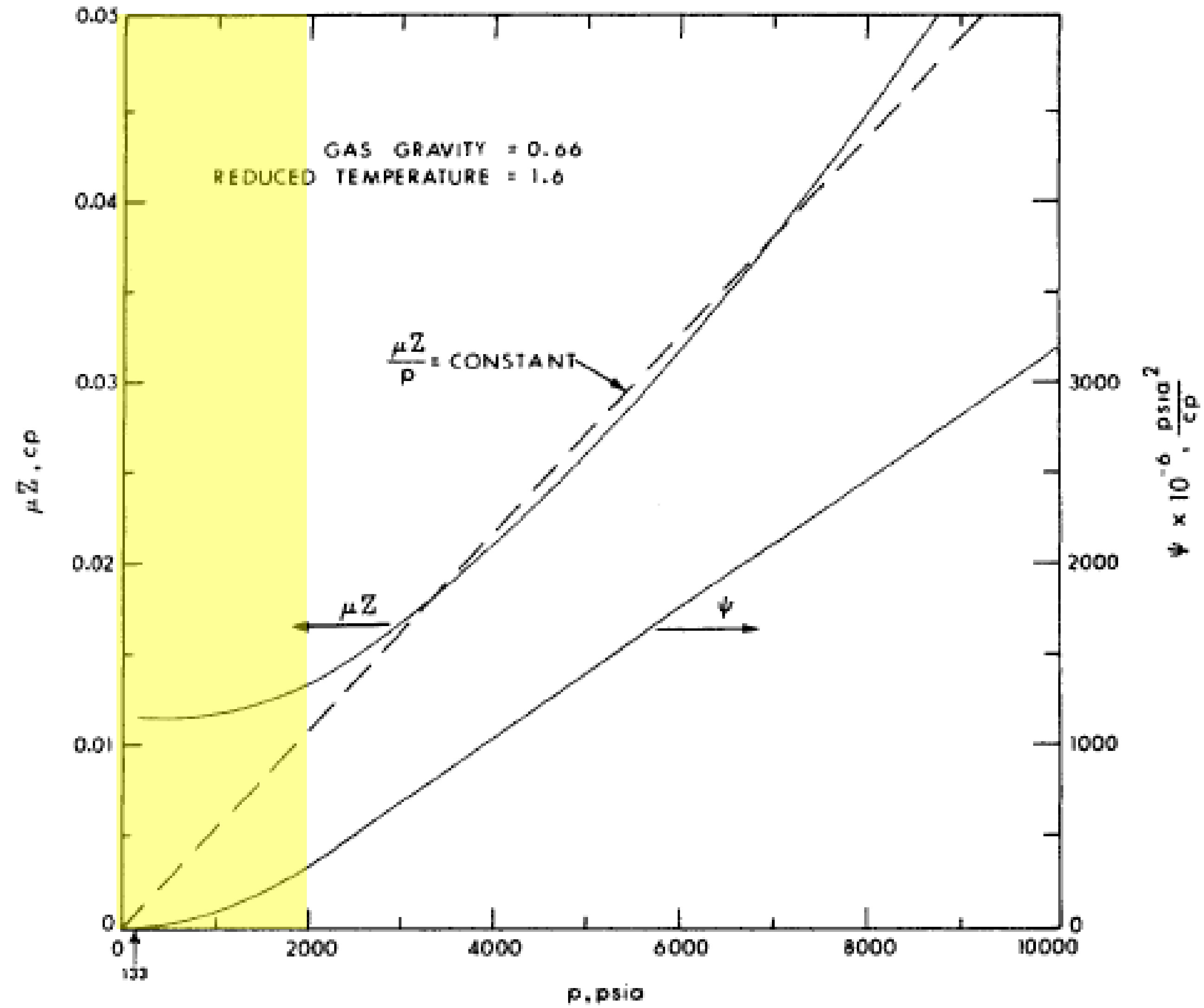


FIGURE 2-3. VARIATION OF ψ AND μZ WITH PRESSURE

From Wattenbarger (1967, p. 99)

Gas Flow Governing Equations

The equations for the "pressure," "pressure-squared," and "pseudo-pressure" treatments already presented may be combined into one general equation of the form

$$\nabla^2\Phi = \frac{1}{\kappa}\frac{\partial\Phi}{\partial t} \qquad (2\text{-}48)$$

where Φ and κ have the following interpretations for the different cases,

	Φ	κ
pressure case	p	$\frac{k}{(\phi\ \bar{\mu}\ \bar{c})}$
pressure-squared case	p^2	$\frac{k}{(\phi\ \bar{\mu}\ \bar{c})}$
pseudo-pressure case	ψ	$\frac{k}{(\phi\ \mu_i c_i)}$