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Hybrid multiple attribute group decision-making for power system restoration

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A B S T R A C T

Due to deregulated power industry, distributed power generation, aging infrastructure and many other factors, the modern society is exposed to higher blackout risks. Decision-making optimization is indispensable for ensuring fast and secure power supply restoration to end users. As an important stage of power system restoration, backbone-network reconfiguration is necessary to re-establish the skeleton network and restore loads. Backbone-network reconfiguration after a large-scale outage is influenced by many factors about system safety and restoration speed. In order to evaluate candidate restoration schemes, multiple types of attributes including crisp data, fuzzy numbers, interval numbers and linguistic terms are employed. An extended VIKOR method is proposed to provide compromise solutions considering hybrid attributes. The method can reflect the vagueness and uncertainties in practical restoration problems, and avoid too much fuzzification. Different forms of preference relations and maximum deviation model are integrated by the minimum relative entropy to determine combined weights of attributes. Sensitivity analysis on the weights provides efficient guidelines for decision makers. Finally, an actual power system demonstrates the basic features of the developed method. It is more reasonable and creditable to consider multiple types of information and unsatisfactory attributes in decision-making.

1. Introduction

In modern society, the increasing demand for reliable power supply makes dynamic behaviors of power systems more complicated. Especially in the market environment, power systems are pushed close to critical operating limits. In recent years, some widespread blackouts have occurred all over the world, including the well-known outages in North America and Canada (2003), Japan (2011) and India (2012) (Feltes & Grande-Moran, 2014; Liu, Hou, Liu, & Podmore, 2014). The blackouts make catastrophic impact on the social life and labor. EPRI has estimated that the US economy is losing between US $104 billion and US $164 billion a year due to outages across all business sectors (Liu, You, Chen, & Fan, 2014). The restoration reports revealed blackouts may be caused by many different factors such as natural disasters, management shortcomings and terrorist attacks. It is unrealistic and impossible to avoid all outages because some factors are beyond the control (Liscouski & Elliot, 2004). With the interconnection and expansion of power systems, many new problems appear, some existing methods are improper for the new problems (Yang, Zhao, & Liu, 2014). Novel techniques and equipment should be developed with the development of power systems. The blackouts seem consistent with the self-organized criticality (SOC). SOC-like dynamics may influence the global complex dynamic behaviors of power systems greatly (Carreras, Newman, Dobson, & Poole, 2004). According to theoretical analysis and actual operation experiences, probability of blackouts can be decreased by applying novel technologies, improving management and optimizing network structures (Liu & Gu, 2007; Qu & Liu, 2012; Ye & Liu, 2013). Nevertheless, blackout is inherently inevitable because of unforeseen circumstances and increasing complication of power systems (Qu & Liu, 2012).

The impact of a blackout increases exponentially with the restoration duration, fast and safe power system restoration is necessary because of catastrophic results of blackouts (Adibi, Borkoski, & Kafka, 1987). Power system restoration is generally implemented in three phases: black-start, backbone-network reconfiguration and load restoration (Lin, Wen, Huang, & Zhou, 2009; Qu & Liu, 2012). Backbone-network reconfiguration is responsible for reestablishing skeleton network of power systems and making full preparation for load restoration. It can be described as a multivariable, multi-objective, multistage and combinatorial optimization

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problem (Lindenmeyer, Dommel, & Adibi, 2001; Shin, Kim, Kim, Choo, & Singh, 2004). There are no known mathematical methods for solving such a NP-complete problem exactly in polynomial time (Toune, Fudo, Genji, Fukuyama, & Nakanishi, 2002). In order to speed up restoration without violating security constraints, many methods have been addressed. Expert system (Park & Lee, 1995) has promising prospects of application, but the establishment and maintenance of knowledge base is a key problem for the large-scale power systems. The change of expert rules may lead to errors and conflicts, the generality and learning ability of expert systems is unsatisfactory. Case based reasoning (CBR) (Islam & Chowdhury, 2001) is dependent on typical scenarios. The bottlenecks of CBR are the small probability of blackouts and difficulty of maintenance. Besides, mathematical programming (Nagata, Sasaki, & Yokoyama, 1995) is advantageous to obtain an optimal solution, but the application is rare due to its sheer difficulty, huge solution space and long execution time. Some computational intelligence algorithms like genetic algorithm (Dong-Joon, Jin-O, Tae-Kyun, Jin-Boo, & Singh, 2004), artificial neutral network (Bretas & Phadke, 2003) and fuzzy theory (Hsiao & Chien, 2000) are also introduced into restoration field. However, the optimality of such methods cannot be guaranteed, and some specific aspects need to be improved for different algorithms respectively, for example the parameters in particle swarm optimization need further researches. Petri net, which is straightforward and highly effective in small systems, is also a popular algorithm for restoration, but it is a little too complex for large systems (Hong-Tzer & Chao-Ming, 2002). The verification of constraints and disposal of uncertainties need further improvements.

In addition, more algorithms like multi-agent (Nagata & Sasaki, 2002) and decision support system (DSS) (Hou et al., 2011) have been proposed for backbone network reconfiguration. DSS is an efficient tool for restoration when restoration time is limited, operators are unfamiliar with decision situations, or the conditions are different from predetermined restoration plans. As the core function of DSS, optimization of decision-making is indispensable (Liu et al., 2014). Most of the abovementioned researches produce restoration plans for the whole system in one time, the restoration paths and sequences are optimized by multiple objective optimization models. However, such methods are generally time-consuming and inflexible, and the restoration plan must be reproduced if some equipment is found unavailable. In real applications, a step-by-step restoration strategy is generally implemented. The restoration goals of each restoration step should be well defined for an actual restoration plan. However, there may be several different restoration goals in each step, many factors influence restoration from different aspects. Different attributes need to be expressed by different forms of variables, the evaluation of candidate restoration schemes is a typical hybrid multiple attribute decision-making (MADM) problem. In MADM, different attributes are generally conflicting, for example, restoration duration may be prolonged to guarantee the system security. Thus, a compromise solution is developed to make a trade-off among different criteria.

Most researches on MADM in restoration focus on the phase of black-start (Lin et al., 2009; Zeng, Lin, Wen, & Ledwich, 2012). Studies on the evaluation of backbone-network reconfiguration schemes are relatively rare. A hybrid MADM method based on VIKOR (in Serbian: Visekriterijumska Optimizacija I Kompromisno Rešenje) is utilized in this paper to implement effective evaluation for backbone-network reconfiguration. Mathematical programming method is used to integrate different forms of subjective preference relations. Combined weights are used to provide more reasonable relative importance of attributes. Euclidean distance is utilized to measure the difference between attributes and alternatives. Considering multiple types of attributes and uncertain information is relevant in the assessment and decision-making of restoration.

The remainder of the paper is structured as follows. Section 2 describes the establishment of candidate restoration schemes and attributes. Section 3 discusses the proposed decision-making method. Section 4 demonstrates the feasibility and practicability of the method with an actual power system. Finally, Section 5 concludes all this paper.

2. Establishment of MADM models

In order to implement MADM in restoration, the candidate restoration schemes and values of attributes should be identified. Restoration schemes are based on restoration strategies. Identification of attributes should consider both the restoration process and the nature of attributes.

2.1. Establishment of candidate restoration schemes

In order to accelerate restoration process, the whole power system is sectionalized into several subsystems in general (Gomes, de Lima, & de Padua Guarini, 2004; Wu & Monticelli, 1988). Therefore the whole system can be restored simultaneously in each subsystem after blackouts. The sectionalization is based on distribution of black-start units, network structure and management compass of power utilities.

The determination of restoration schemes consists of two parts: (1) determination of restoration goals; (2) restoration path searching. After the restoration goals are determined, they can be regarded as the destination node, the energized network is the initial node. Consequently the shortest restoration path can be obtained with Dijkstra algorithm (Johnson, 1973). In each subsystem, the restoration goals are determined by a three-stage restoration strategy (Liu, Sun, & Wang, 2015). In different stages different restoration goals are focused on. In the first restoration stage, restoration of large units is of the most importance; in the second stage, more attention is paid to restoring important substations; in the third stage, synchronization of subsystems and interconnection assistance is the most important. There are no strict lines between these three stages. In the second and third stages, if some unit which cannot be restored in the first stage satisfies startup conditions, it should be started up preferentially. The main principle of the three-stage restoration strategy is that restoration goals are decided by the restoration process.

Dijkstra algorithm is implemented for restoration path searching in the paper. The weights of edges can be defined according to the requirements of restoration, such as reactive charging power of transmission lines, restoration time of equipment and length of transmission lines. For the feasibility of candidate restoration schemes, security checking needs to be implemented in each restoration step. If the security demand is not satisfied, network parameters can be adjusted. The candidate restoration schemes will be discarded if they are still unsatisfactory after adjustments.

For overvoltage and reactive power imbalance, they can be improved by adjusting generator terminal voltages, switching reactive compensation equipment, adjusting the load and changing transformer tap positions. The overvoltage can be improved by reducing steady voltage level. Transient voltage dip caused by startup of auxiliary equipment can be improved by adjusting the startup sequence of auxiliary equipment and raising the voltage of receiving end properly. The out-of-limit available transfer capability (ATC) can be resolved by adjusting the load and outputs of generators. The frequency dip caused by load pick-up can be improved by adjusting load restoration. However, if the number of candidate restoration schemes which need adjustments is relatively large, weights of edges for path searching can be changed.

The procedure of determination of candidate restoration schemes is illustrated in Fig. 1.

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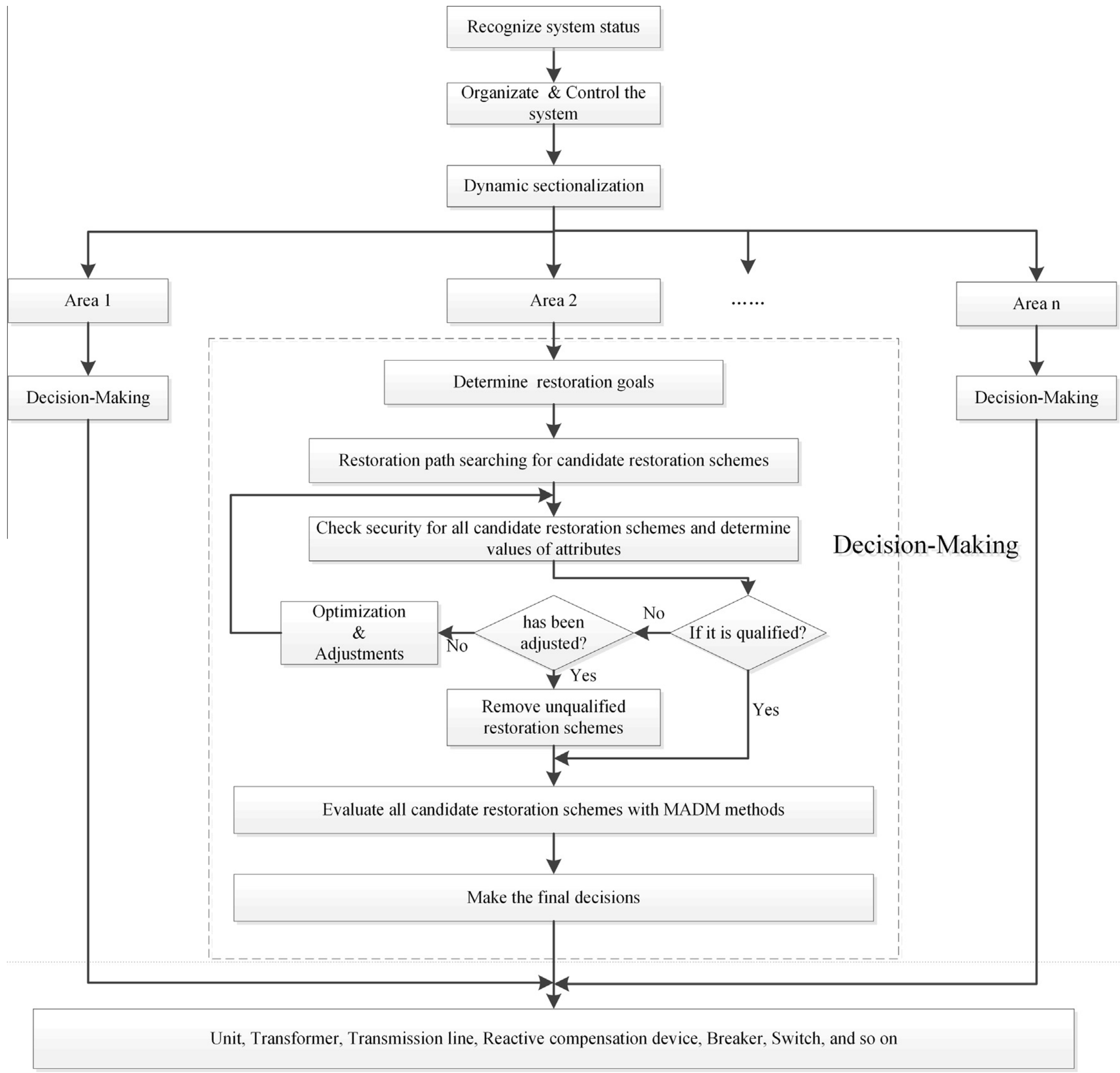

Fig. 1. Determination of candidate restoration schemes.

2.2. Determination of attributes

Reliable evaluation index system is the basis of an effective evaluation. The establishment of evaluation index system needs the guidance of scientific theories and practical problems. It should reflect the goals, constraints and desired results of each restoration stage. The attributes need to be identified through literature review, widespread investigation and consultation with experts (Zhang, Wu, Feng, & Yu, 2011). In decision-making, all factors representing characteristics of alternatives can be used as attributes. For backbone-network reconfiguration, attributes can adopt indexes related to system security, restoration speed and restoration benefits. The security problems mainly include switching overvoltage, sustained power frequency overvoltage, power flow of branches, frequency fluctuation, transient voltage dip and etc. The restoration speed depends on startup time of unit, unit ramp rate, importance of generators, restoration time of equipment and so on. The restoration benefits are related to losses and negative impact of blackouts.

In restoration, the abovementioned attributes can be classified into three types: (1) crisp data, some attributes like switching overvoltage can be determined through field experiment, simulations or computation; (2) uncertain variables, due to the complexity and uncertainty of objective things, some attributes like restoration duration cannot be quantified with crisp values, only the range of the attributes can be given; (3) linguistic terms, some attributes like importance of node can be expressed with linguistic terms to ease the burdens of decision makers (DMs). The type of attributes needs to reflect the nature of the problem and requirements of DMs. The attributes are necessary to be attainable and measurable. Three major solutions for hybrid attributes are shown in Table 1.

The nature of the data influences the treatment of data. For restoration, some data like the voltage of buses can be obtained directly from EMS (Energy Management System), WAMS (Wide Area Measurement System), GIS (Geographic Information System), etc. Such data can be modeled with crisp numbers. While many data needs to be obtained by simulation or computation with existing data in the database, for example, sustained power frequency overvoltage can be obtained by the simulation or calculation of power flow. For data like restoration duration, it should theoretically be quantified by crisp values. However, it is impossible to determine the exact restoration time for uncertain and complex factors in power systems. If treated as crisp data, the uncertainties of equipment are ignored, which means

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Table 1
Solutions for hybrid attributes.

Solutions	Characteristics
Quantify uncertain variables and linguistic terms with mathematical algorithms and complex network theory (Liu & Gu, 2007).	(1) Different results may be obtained with different qualification algorithms (2) Loss of information if uncertain variables are simplified by crisp data (3) Ignorance of complexity of human cognition and uncertainty of objective things
Express linguistic terms with uncertain variables, and translate uncertain variables to crisp values (Liu et al., 2014)	(1) No mature definition of expectation or center for different types of variables (2) Loss of information (3) Various qualification definitions
Regard crisp values as special uncertain variables (Opricovic, 2011)	(1) Computation is complicated (2) It is not sure reliability of decision-making is improved (3) Too much fuzzification does not imply better modeling of reality, it can be counterproductive

information losses. For attributes like importance of generating units, such data can be quantified with crisp data by the complex network theory. However, different results obtained by different methods may confuse DMs, and too much time may be needed. Different methods of treatment bring different errors. The treatment methods of data should be selected according to decision situations, nature of data and requirements of DMs.

2.3. Measurement of attributes

2.3.1. Normalization of decision matrix

In the decision-making process, attributes are generally conflicting and they are expressed with different units and on different scales. Normalization is made column-wise as Table 2 to make attributes conform to a norm for convenient comparison. In the normalized decision matrix F, the larger f_{ij} is, the better scheme a_i performs in the attribute j. The linear normalization does not depend on the unit of attributes, and the non-linear normalized values could be different for different evaluation unit of a particular attribute (Opricovic & Tzeng, 2004). Such a linear normalization can guarantee that different types of attributes have similar principles of normalization. It is easy to implement and understand, and it can keep the initial ratio of the attributes. For other linear normalization like $r_{ij} = d_{ij}/\sum_i d_{ij}$ or $r_{ij} = (d_{ij} - \min(d_{ij}))/(\max(d_{ij}) - \min(d_{ij}))$ (the decision matrix $D = (d_{ij})_{m\times n}$), it is hard to adjust for interval numbers and fuzzy numbers, the relationship between the elements of interval numbers and fuzzy numbers may be changed. It is hard to judge the influence of such treatment on the decision-making.

2.3.2. Measurement of distances

The decrease in information losses depends on the nature of the variables. However, the data processing can also lose some information. For example, when the triangle fuzzy numbers are transformed into crisp data, at least the distribution and uncertainty of the data are ignored. In many researches (Vinodh, Varadharajan, & Subramanian, 2013; Yeonjoo & Eun-Sung, 2013), attributes are transformed into the same type, then some distance metric is used to measure the difference. In this paper, attributes can be expressed with different types including crisp data, interval numbers, fuzzy numbers and linguistic variables. As long as proper distance metric is defined and the comparability of distance between different attributes is guaranteed, a hybrid decision matrix can be directly utilized with different types of information without attributes transformation. Euclidean distance is utilized to avoid attribute transformation in the process of data processing. In theory, any reasonable distance metric can be used to measure the distance. Choosing Euclidean distance is not only because it is common-used and easy-processed. With this distance metric, the decision-making process can be consistent with security checking and other subroutines of restoration. Euclidean distance is a visual and clear definition for distance metric, and it can be easily extended to different types of attributes. Euclidean distance conforms to the distance definition of the VIKOR method and the objective weights.

For crisp values a, b, interval numbers $[a^L, a^U]$, $[b^L, b^U]$, and fuzzy numbers $[a^L, a^M, a^U]$, $[b^L, b^M, b^U]$, the distance is expressed as follows (Aghajani Bazzazi, Osanloo, & Karimi, 2011):

$$D(A,B) = \begin{cases} |a-b| \\ \sqrt{\frac{1}{2}[(a^L-b^L)^2+(a^U-b^U)^2]} \\ \sqrt{\frac{1}{3}[(a^L-b^L)^2+(a^M-b^M)^2+(a^U-b^U)^2]} \end{cases} \quad (1)$$

As for linguistic variables, Euclidean distance is obtained after they are transformed into other types of variables. Corresponding relationship is shown in Table 3.

3. Multiple attribute decision-making method

Multiple attribute decision-making is concerned with evaluating a finite number of decision alternatives under a finite number of criteria or attributes (Zanakis, Solomon, Wishart, & Dublish, 1998). Research contents of MADM include selection of evaluation algorithm, weighting method of attributes and sensitivity analysis (Munda, 2004). After the evaluation index system is established, a MADM method is selected for multicriteria aggregation. Except for some particular methods, weighting method of attributes is necessary for aggregation and decision-making process (Choo, Schoner, & Wedley, 1999). Sensitivity analysis is not essential for MADM, but it can provide the opportunity to analyze the stability of results.

3.1. Selection of MADM method

The diversity of MADM methods provides multiple optional choices. However, it may be seen as a drawback. It is hard to judge whether one method is more suitable and creditable than another for a specific decision-making problem. Different decision makers

Table 2
Normalization of attributes.

	Cost attribute	Benefit attribute
Crisp data (a_{ij})	$\frac{\min_i a_{ij}}{a_{ij}}$	$\frac{a_{ij}}{\max_i a_{ij}}$
Interval number $\left(\left[a_{ij}^L, a_{ij}^U\right]\right)$	$\left[\frac{\min_i a_{ij}^L}{a_{ij}^U}, \frac{\min_i a_{ij}^L}{a_{ij}^L}\right]$	$\left[\frac{a_{ij}^L}{\max_i\ a_{ij\ i}^U}, \frac{a_{ij}^U}{\max_i\ a_{ij\ i}^U}\right]$
Fuzzy number $\left(\left[a_{ij}^L, a_{ij}^M, a_{ij}^U\right]\right)$	$\left[\frac{\min_i a_{ij}^L}{a_{ij}^U}, \frac{\min_i a_{ij}^L}{a_{ij}^M}, \frac{\min_i a_{ij}^L}{a_{ij}^L}\right]$	$\left[\frac{a_{ij}^L}{\max_i a_{ij}^U}, \frac{a_{ij}^M}{\max_i a_{ij}^U}, \frac{a_{ij}^U}{\max_i a_{ij}^U}\right]$

Table 3
Corresponding relationship between different variables.

Linguistic variables	Interval number	Fuzzy number
Low (L)	[0,0.2]	[0,0.1,0.2]
Medium low (ML)	[0.2,0.4]	[0.2,0.3,0.4]
Medium (M)	[0.4,0.6]	[0.4,0.5,0.6]
Medium High (MH)	[0.6,0.8]	[0.6,0.7,0.8]
High (H)	[0.8,1.0]	[0.8,0.9,1]

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will always disagree on which methods are the most appropriate and valid (Loken, 2007). Choosing MADM methods can be also seen as a MADM problem, but it is important to avoid the vicious circle of using an MADM tool to choose an MADM method (Guitouni & Martel, 1998).

As for power system restoration, the decision-making situations should be analyzed in detail and various MADM methods need to be compared in simplicity, assumptions, application conditions, robustness and trustworthiness to choose a proper one. In power system restoration, the time is pressing and the decision makers are under tremendous pressure. For large power systems, there are a large number of alternatives and attributes. The evaluation of candidate restoration schemes is a dynamic decision-making problem. In each restoration step, restoration goals are determined by the network status and restoration strategies dynamically. In different stages and even different steps of restoration, different attributes need to be selected for restoration. The weights of attributes should be adjusted according to the restoration process and requirements of DMs. For example, in the early stage of restoration, reliability of units is more important. However, in the late restoration stage, since network is much stronger, restoration duration needs more attention to reduce losses. The restoration schemes should be adjusted or discarded dynamically if unavailable restoration path is found. Time variation of factors are considered in the real application. For instance, in order to decrease restoration time, the unit which is close to its critical time constraint needs to be set a larger weight to avoid cold start.

The MADM method should be selected according to the decision-making situation and the needs of decision makers (Kurka & Blackwood, 2013). For the complex decision problem of restoration, a systematic but easy method is more proper, and the method should be able to deal with large quantities of alternatives and attributes.

AHP (Analytic Hierarchy Process) is a popular MADM method for its rational hierarchy structure and clear logic relations (Ahmad, Saman, Mohamad, Mohamad, & Awang, 2014). Because the large number of pairwise comparison and alternatives will both influence the validity and consistency of AHP (Loken, 2007), it becomes very difficult to use for large power system restoration. In the dynamic decision-making process of power system restoration, the criteria are changing dynamically in different restoration stages. Therefore the hierarchy structure may be too complex for the limited restoration time, and it must be adjusted as the restoration proceeds. With the expansion of restored power systems, the number of alternatives may be too huge for AHP. In many researches, AHP is often used for weighting attributes to combine hybrid MADM methods but not for ranking the alternatives (Chang, 2014; Tyagi, Kumar, & Kumar, 2014).

TOPSIS (Technique for order preference by similarity to an ideal solution) is a good choice because it is easy to assimilate and implement (Behzadian, Khanmohammadi Otaghsara, Yazdani, & Ignatius, 2012). The method is intuitively appealing for its visualization, at least for two dimensions. Considering both the best and worst solutions makes TOPSIS reasonable and rational. It performs quite well in the rank reversals (Mokhtarian, Sadi-nezhad, & Makui, 2014). However, the relative importance of the distances from the negative and positive ideal solutions is not considered in the method, and the definition of the best alternative is not always true (Opricovic & Tzeng, 2004).

For outranking methods like ELECTRE and PROMETREE which can rank alternatives without normalization, the DMs' preferences are expressed in a realistic way by recognizing hesitations in their mind, and uncertainties can be considered in various ways (Chatterjee, Athawale, & Chakraborty, 2009; Opricovic & Tzeng, 2007). However, such methods may cost much time for making pairwise comparisons between each two alternatives. The preference thresholds and functions for each criterion may also be a tough and subjective task for DMs (Anojkumar, Ilangkumaran, & Sasirekha, 2014). The outranking methods are normally not used for selection, but very suitable for screening process in the initial step of decision-making (to categorize alternatives into acceptable or unacceptable) (Loken, 2007). Opricovic and Tzeng (2007) made a comparative analysis of VIKOR and outranking methods, PROMETHEE with a linear preference function, gives the same results as VIKOR, with measure S representing group utility. Results by ELECTRE II, with linear "surrogate" criterion functions, are relatively similar to the results by VIKOR.

Because of the demerits of abovementioned methods, VIKOR is adopted in this paper. VIKOR is a simple but systematic MADM method, and it is advantageous particularly in a situation where the DM is not able or does not know how to describe his/her preferences at the beginning of system design (Opricovic & Tzeng, 2004). With respect to restoration, due to various alternatives in each restoration step, unsatisfactory attributes affect the selection remarkably. An unsatisfactory attribute means the attribute is close to the limit, the restoration duration may be prolonged or the benefit of the restoration scheme may be unsatisfactory, it is unnecessary for the DMs to risk choosing such an alternative. For most existing MADM methods applied in restoration, the alternatives are ranked based on utility theory and simple additive weighting (SAW) for the limited restoration time and decision makers (DMs) under tremendous pressure (Lin et al., 2009; Zeng et al., 2012). Up to now, the MADM method incorporating specific techniques to avoid compensation has not been published in restoration decision-making. In such cases, the bad performance of some attribute can be made up by other attributes. Even if some attribute is very close to its safe limit, the restoration scheme is probably chosen because other attributes play a positive role. However, in real application, such a restoration scheme is likely to be infeasible because of uncertain factors. Delay and even failure of restoration may occur in such situations. Hence, VIKOR is chosen because it considers not only maximum group utility, which reflects the best performance of all criteria, but also individual regret, which represents the most unsatisfactory attribute. The DMs can adjust the weights of group utility and individual regret according to the various decision-making situations dynamically.

3.2. VIKOR method

VIKOR is proposed by Opricovic in 1998, and it originates from L_p-metric in compromise programming. It has been applied in areas including engineering design, economy, management and military (Tzeng, Lin, & Opricovic, 2005). The implementation of VIKOR has been increasing in the recent years. Kang and Park (2014) measured customer satisfaction in mobile service with VIKOR, and they made a sensitivity analysis on the parameter v. Chang and Hsu (2011) classifies land subdivisions by assessing environmental characteristics and vulnerabilities. Chang and Hsu (2009) used VIKOR to rank land-use restrictions. For the fuzzy environment, there are also some researches. Liu, You, You, and Shan (2015) used a fuzzy VIKOR for failure mode and effects analysis, triangular fuzzy numbers are preferred to express linguistic evaluations. Rostamzadeh, Govindan, Esmaeili, and Sabaghi (2015) transformed linguistic terms to fuzzy numbers for VIKOR in evaluation of green supply chain management practices. Kaya and Kahraman (2010) transformed the fuzzy numbers to crisp data for renewable energy planning. Tadić, Zečević, and Krstić (2014) combines DEMATEL, ANP and VIKOR methods in a fuzzy context. Mokhtarian et al. (2014) used fuzzy VIKOR on interval valued fuzzy numbers for facility location selection problems. You, You, Liu, and Zhen (2015) used linguistic VIKOR method for supplier selection,

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the attributes are expressed with 2-tuple linguistic variables. Mousavi, Jolai, and Tavakkoli-Moghaddam (2013) used a stochastic VIKOR to evaluate and rank probability distributions for each alternative. Mousavi, Torabi, and Tavakkoli-Moghaddam (2013) ranked alternatives with a fuzzy value Q_i, Q_i with higher mean and lower spread is preferred. For hybrid attributes in VIKOR, the studies are relatively few, Aghajani Bazzazi et al. (2011) used Euclidean distances and VIKOR to deal with hybrid attributes, but it is not extended to group decision-making situations, the weights are obtained by AHP and information entropy.

VIKOR provides compromise solutions based on the closeness between alternatives and the ideal solution. The compromise means an agreement established by mutual concession (Opricovic & Tzeng, 2004). L_p-metric aggregate function is adopted in the overall evaluation:

$$L_{p,j} = \left\{ \sum_{i=1}^{n} \left[\frac{w_i(F_i^+ - f_{ij})}{F_i^+ - F_i^-} \right]^p \right\}^{1/p} \tag{2}$$

where w_i is the weight of attribute i; F_i^+, F_i^- are respectively the PIS (positive ideal solution) and NIS (negative ideal solution); $L_{p,j}$ denotes the distance between PIS and alternative a_j. In the method, $L_{1,j}$ (S_i in (4)) and $L_{\infty,j}$ (R_i in (5)) are utilized for ranking. The detailed stepwise procedure of VIKOR is as follows:

Step 1: Prepare the data including normalized decision matrix, weights of attributes and types of attributes for decision-making.

Step 2: Determine PIS and NIS, respectively:

$$F_i^+ = \begin{cases} \max\limits_i f_{ij}, & j \in N_1 \\ \left[\max\limits_i f_{ij}^L, \max\limits_i f_{ij}^U\right], & j \in N_2 \\ \left[\max\limits_i f_{ij}^L, \max\limits_i f_{ij}^M, \max\limits_i f_{ij}^U\right], & j \in N_3 \end{cases}, \quad F_i^- = \begin{cases} \min\limits_i f_{ij}, & j \in N_1 \\ \left[\min\limits_i f_{ij}^L, \min\limits_i f_{ij}^U\right], & j \in N_2 \\ \left[\min\limits_i f_{ij}^L, \min\limits_i f_{ij}^M, \min\limits_i f_{ij}^U\right], & j \in N_3 \end{cases} \tag{3}$$

where N_1, N_2, N_3 denote the set of crisp data, interval numbers and fuzzy numbers, respectively.

Step 3: Calculate the values of S_i, R_i and Q_i by the relations:

$$S_i = \sum_{j=1}^{n} w_j \frac{D(F_i^+, f_{ij})}{D(F_i^+, F_i^-)} \tag{4}$$

$$R_i = \max \left(w_j \frac{D(F_i^+, f_{ij})}{D(F_i^+, F_i^-)} \right) \tag{5}$$

$$Q_i = v \frac{S_i - \min S_i}{\max S_i - \min S_i} + (1 - v) \frac{R_i - \min R_i}{\max R_i - \min R_i} \tag{6}$$

where S_i is the group utility of alternative a_i; R_i is the individual regret of the worst index of a_i. $v \in [0, 1]$ is the weight of majority criteria and $(1 - v)$ is the weight of individual regret. Commonly $v = 0.5$ is set to achieve the tradeoff between majority rules and individual regret (Aghajani Bazzazi et al., 2011; Anojkumar et al., 2014). For S_i, R_i and Q_i, the less they are, the better a_i performs in decision-making.

Step 4: Rank the alternatives by ascending order according to S_i, R_i, Q_i, respectively.

Step 5: Propose a compromise solution. The compromise solution is $a^{(1)}$, which is the best-ranked solution in the sequence of Q_i, if the following two conditions are satisfied:

C1-"Acceptable advantage": $Q(a^{(2)}) - Q(a^{(1)}) \geqslant 1/(m-1)$, $a^{(2)}$ is the second-best ranked solution by the sequence of Q_i, and m is the number of alternatives.

C2-"Acceptable stability in decision-making": $a^{(1)}$ is also best ranked by S_i or R_i.

If condition C1 is not satisfied, $a^{(1)}, a^{(2)}, \ldots, a^{(r)}$ are all compromise solutions, where $Q(a^{(r)}) - Q(a^{(1)}) \geqslant 1/(m-1)$. If C2 is not satisfied, $a^{(1)}$ and $a^{(2)}$ are both compromise solutions.

In actual decision-making, condition C1 is not always satisfied. It means $a^{(1)}$ is not distinctly more advantageous than others, the final decision can be made by expert experiences. The three sequences of S_i, R_i, Q_i can be comprehensively considered to choose a proper restoration scheme. The threshold of condition C1 and the value of v can be adjusted for the new decision-making process to rank candidate restoration schemes.

3.3. Weights of attributes

Determination of weights of attributes is critical but challenging for MADM. These weights in MADM do not have a clear economic significance, but their use provides the opportunity to model the actual decision-making (Opricovic & Tzeng, 2004). Considering advantages and disadvantages of both subjective and objective weighting method, combined weighting method is adopted in this paper.

3.3.1. Subjective weighting method

Preference relations are frequently used to express DMs' preference information (Chen & Niou, 2011; Wei, 2010). In group decision-making, due to different educational background and domain knowledge, DMs may provide different subjective preferences on attributes with different types of preference relations under different decision situations. It is necessary to integrate different subjective preference relations with a combined mathematical model (Wang & Parkan, 2006). Multiplicative and fuzzy preference relations are two common means to express subjective preferences.

Definition 1. A preference relation P on the set X is characterized by a function $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees provided by the DM for each pair of alternatives (Xu, 2007).

Definition 2 Satty, 1980. A multiplicative preference relation P on the set X is defined as a reciprocal matrix $B = (b_{ij})_{n \times n} \subset X \times X$, under the condition:

$$b_{ij} b_{ji} = 1, \quad b_{ii} = 1, \quad b_{ij} > 0 \quad \text{for all } i, j = 1, 2, \ldots, n \tag{7}$$

where b_{ij} is the ratio of preference intensity of attribute i to attribute j. $b_{ij} = 1$ indicates indifference between attribute i and j. $b_{ij} > 1$ indicates attribute i is preferred to attribute j, the stronger the preference intensity of attribute i over attribute j, $b_{ij} = 9$ means attribute i is extremely preferred to attribute j (Xu, 2007).

Definition 3. A fuzzy preference relation P on the set X is defined as a complementary matrix $B = (b_{ij})_{n \times n} \subset X \times X$, under the condition:

$$b_{ij} + b_{ji} = 1, \quad b_{ii} = 0.5, \quad b_{ij} \geqslant 0 \quad \text{for all } i, j = 1, 2, \ldots, n \tag{8}$$

where $b_{ij} = 0.5$ implies indifference between attribute i and j. $b_{ij} > 0.5$ indicates attribute i is preferred to attribute j, the stronger the preference intensity of attribute i over attribute j, $b_{ij} = 1$ means attribute i is extremely preferred to attribute j (Tanino, 1984).

Xu (2007) introduced many other preference relations, and they can be transformed into a multiplicative preference relation with different models. For simplicity, only multiplicative preference

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relations and fuzzy preference relations are utilized in this paper. However, the method of the paper can be easily extended to different preference relations.

Subjective weights can be obtained by Eigenvalue Method (EM) from preference relations. Without loss of generality, it is assumed that k_1 decision makers provide multiplicative preference relations whereas k_2 decision makers prefer fuzzy preference relations. It is hoped that the two kinds of preference relations should be as consistent as possible. Therefore the integrated optimization model is constructed:

$$\min\ F = \left[\sum_{l=1}^{k_1}\left(\alpha_{(l)}\sum_{i=1}^{n}\left|\varepsilon_i^{(l)}\right|^p\right) + \sum_{m=1}^{k_2}\left(\beta_{(m)}\sum_{j=1}^{n}\left|r_j^{(m)}\right|^p\right)\right]^{\frac{1}{p}}$$
$$s.t.\begin{cases}(A^{(l)}-nI)W - E^{(l)} = \mathbf{0}, & l=1,2,\ldots,k_1\\ (H^{(m)}-nI)W - R^{(m)} = \mathbf{0}, & m=1,2,\ldots,k_2\\ e^TW=1\\ W \geqslant 0\end{cases} \quad (9)$$

where n denotes the number of attributes; I is an $n\times n$ unit matrix; $A^{(l)}$ represents multiplicative preference relations; $H^{(m)}$ is obtained from fuzzy preference relations $P^{(m)}$ and $h_{ij}=p_{ij}/p_{ji}$; $E^{(l)} = (\varepsilon_1^{(l)},\varepsilon_2^{(l)},\ldots,\varepsilon_n^{(l)})^T$ and $R^{(m)} = (r_1^{(m)},r_2^{(m)},\ldots,r_n^{(m)})^T$ are deviation vectors; $W=(w_1,w_2,\ldots,w_n)^{\mathrm{T}}$ is the subjective weight vector; $e=(1,\ldots,1)^T$; $\alpha_{(l)}\geqslant 0,\ l=1,2,\ldots,k_1, \beta_{(m)}\geqslant 0,\ m=1,2,\ldots,k_2$ are respectively the weights of DMs satisfying $\sum_{l=1}^{k_1}\alpha_{(l)}+\sum_{m=1}^{k_2}\beta_{(m)}=1$; $p>0$ is a parameter on deviation norm, theoretically p can take any positive number. However, for decreasing computation burden, $p=1,2,\infty$ is set practically.

(1) $p=1$

Let $p=1$ in (9), the optimization model can be resolved by Linear Programming (LP):

$$\min\ F = \sum_{l=1}^{k_1}\alpha_{(l)}e^T(E^{(l)+}+E^{(l)-}) + \sum_{m=1}^{k_2}\beta_{(m)}e^T(R^{(m)+}+R^{(m)-})$$
$$s.t.\begin{cases}(A^{(l)}-nI)W - E^{(l)+} + E^{(l)-} = \mathbf{0}, & l=1,2,\ldots,k_1\\ (H^{(m)}-nI)W - R^{(m)+} + R^{(m)-} = \mathbf{0}, & m=1,2,\ldots,k_2\\ e^TW=1\\ W, E^{(l)+}, E^{(l)-}, R^{(m)+}, R^{(m)-} \geqslant 0\end{cases} \quad (10)$$

where $E^{(l)+} = \left(\varepsilon_1^{(l)+},\varepsilon_2^{(l)+},\ldots,\varepsilon_n^{(l)+}\right)^T$, $E^{-} = \left(\varepsilon_1^{(l)-},\varepsilon_2^{(l)-},\ldots,\varepsilon_n^{(l)-}\right)^T$, $R^{(m)+} = \left(r_1^{(m)+},r_2^{(m)+},\ldots,r_n^{(m)+}\right)^T, R^{(m)-} = \left(r_1^{(m)-},r_2^{(m)-},\ldots,r_n^{(m)-}\right)^T$, they are generated from the deviation vectors $E^{(l)}$ and $R^{(m)}$:

$$\varepsilon_i^{(l)+} = \frac{\varepsilon_i^{(l)}+|\varepsilon_i^{(l)}|}{2},$$
$$\varepsilon_i^{(l)-} = \frac{-\varepsilon_i^{(l)}+|\varepsilon_i^{(l)}|}{2} \quad i=1,2,\ldots,n,\ l=1,2,\ldots,k_1$$
$$r_i^{(m)+} = \frac{r_i^{(m)}+|r_i^{(m)}|}{2},$$
$$r_i^{(l)-} = \frac{-r_i^{(m)}+|r_i^{(m)}|}{2} \quad i=1,2,\ldots,n,\ m=1,2,\ldots,k_2 \quad (11)$$

(2) $p=2$

Let $p=2$ in (9), the optimization model becomes a Quadratic Programming (QP) problem:

$$\min\ F = W^TGW$$
$$s.t.\begin{cases}e^TW=1\\ W\geqslant 0\end{cases} \quad (12)$$

where $G=\sum_{l=1}^{k_1}\alpha_{(l)}(A^{(l)}-nI)^T(A^{(l)}-nI)+\sum_{m=1}^{k_2}\beta_{(m)}(H^{(m)}-nI)^T(H^{(m)}-nI)$, such a problem can be resolved by Lagrangian function or Mathematical Programming (MP) (Ma, Fan, & Huang, 1999), and $W=G^{-1}e/e^TG^{-1}e$.

(3) $p=\infty$

Let $p=\infty$, the optimization model can be rewritten as a typical LP problem:

$$\min\ F = \sum_{l=1}^{k_1}\alpha_{(l)}\varepsilon^{(l)} + \sum_{m=1}^{k_2}\beta_{(m)}r^{(m)}$$
$$s.t.\begin{cases}-\varepsilon^{(l)}\cdot e \leqslant (A^{(l)}-nI)W \leqslant \varepsilon^{(l)}\cdot e\\ -r^{(m)}\cdot e \leqslant\ (H^{(m)}-nI)W \leqslant r^{(m)}\cdot e\\ e^TW=1\\ W,\varepsilon^{(l)},r^{(m)}\geqslant 0\end{cases} \quad (13)$$

where $\varepsilon^{(l)} = \max_i\left|\varepsilon_i^{(l)}\right|$ and $r^{(m)} = \max_i\left|r_i^{(m)}\right|$.

According to the mathematical models, the computation burden of subjective weighting method is mainly decided by the number of constraints. In real application, the number of decision makers and attributes are generally limited. Therefore it can satisfy the requirements of real-time application. Wang and Parkan (2006) found if $p=2$ is set, the corresponding QP model will be more sensitive to the changes of weights of DMs than Linear Programming models. When $p=2$, the subjective weights can be obtained by matrix operation, the computation burden is reduced. However, if it is difficult for DMs to determine an appropriate value of p, it is suggested that $p=1,2,\infty$ be all set so that the final decision can be made on a majority basis.

3.3.2. Objective weighting method

Objective weighting method is completely based on decision matrices and mathematical algorithms. If the difference of some specific attribute in different alternatives is not obvious, the attribute provides less information in decision-making. Thus the objective weight is naturally small, and vice versa (Xu, 2004). In this paper, the maximum deviation model is utilized:

$$\max\ J = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}w_j''D(f_{ij},f_{kj})$$
$$s.t.\begin{cases}\sum_{i=1}^{n}w_j''^2 = 1 \quad j=1,2,\ldots,n\\ w_j''\geqslant 0\end{cases} \quad (14)$$

where $F=(f_{ij})_{m\times n}$ indicates normalized decision matrix; w_j'' is the objective weight of attribute j, $D(\bullet)$ is the Euclidean distance metric. The meaning of the model is to maximize the difference between different attributes in different alternatives. The objective weights can be obtained by Lagrangian function:

$$w_j'' = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}D(f_{ij},f_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}D(f_{ij},f_{kj})} \quad (15)$$

3.3.3. Combined weighting method

Subjective weights depend on expert experiences. The numerical characteristics and time variation of attributes cannot be reflected. Due to human intervention, the subjective influence on determining subjective weights is inevitable. Objective weights can reflect variations of attributes in real time. However, the generality and interactivity is unsatisfactory, and the computation is generally complex. Because of the ignorance of subjective preferences, objective weights may contradict with actual importance of attributes. Moreover, some knowledge and strategies of restoration are difficult to be expressed in a totally objective manner. For the abovementioned reasons, the combined weights are adopted. The combined weights can reflect subjective preferences and requirements, and the variations of attributes can also be considered. The deficiencies of both weighting methods can be overcome, and the decision-making is more reasonable and reliable (Xu, 2004). In this paper, the combined weights can be obtained by the minimum relative entropy (Vedral, 2002):

$$\min J = \sum_{j=1}^{n} w_j[\ln(w_j/w_j')] + \sum_{j=1}^{n} w_j\left[\ln\left(w_j/w_j''\right)\right]$$
$$s.t. \begin{cases} \sum_{j=1}^{n} w_j = 1 \\ w_j \geqslant 0 \quad j = 1, 2, \ldots, n \end{cases} \tag{16}$$

where w_j', w_j'', w_j are respectively subjective, objective and combined weight of attribute j. The meaning of the model is to minimize the amount of information losses caused by the adoption of combined weights. The solution of the mathematical model is

$$w_j = \frac{\left(w_j' w_j''\right)^{0.5}}{\sum_{j=1}^{n}\left(w_j' w_j''\right)^{0.5}} \tag{17}$$

3.4. Sensitivity analysis

Stability of ranking results is important for DMs. Small changes of weights may alter the initial ranking. Therefore sensitivity analysis is necessary to define the stability intervals for weights of attributes (Opricovic & Tzeng, 2007). The values of weights within the stability interval do not change the results obtained with initial coefficients.

Due to some disturbances, the weight of attribute k changes from its initial value w_k to w_k', and $w_k' = dw_k$. According to the value of w_k, the parameter d varies in the interval $[0, 1/w_k]$. Other weights are modified keeping initial ratios, the coefficient $\phi(d)$ can be obtained from the constraint $dw_k + \phi\sum_{i \neq k}^{n} w_j = 1$, and $\phi(d) = (1 - dw_k)/(1 - w_k)$. The parameter $d \in [d_1, d_2]$ can be determined with the same compromise solutions obtained from the initial weights. The stability interval of the weight of attribute k $[w_k^1, w_k^2]$ is consequently obtained. The mathematical model is as follows:

$$\begin{cases} \tau(Q_1, Q_2, \ldots, Q_m) = \tau(Q_1', Q_2', \ldots, Q_m') \\ w_j' = \frac{1-dw_k}{1-w_k} w_j = \phi w_j & j \neq k \\ w_j' = dw_k & j = k \end{cases} \tag{18}$$

where w_j and w_j' are respectively the weight of attribute j before and after disturbances; $\tau(Q_1, Q_2, \ldots, Q_m)$, $\tau(Q_1', Q_2', \ldots, Q_m')$ are respectively the inversion numbers of sequence Q_i before and after disturbances. For the sequence $(A(1), \ldots, A(n))$ which is composed of n distinct numbers, if $i < j$ and $A(i) > A(j)$, then the pair (i, j) is called an inversion. The number of inversions is defined as the inversion number. In the sequence of Q_i, if $Q_i = Q_j$, $i \neq j$, it indicates the ranking results are unstable and some coefficients can be adjusted for the new decision-making.

Assuming the initial weight is w^0, and its stability interval is $[w^1, w^2]$, $\Delta = \min(w^0 - w^1, w^2 - w^0)$. The sensitivity coefficient is defined as $1/\Delta$. If $(w^1 - w^0)(w^2 - w^0) = 0$, the sensitivity coefficient is defined as positive infinite.

As for the sensitivity analysis on the values of attributes, it can be obtained in a similar way, and the sensitivity analysis on the parameter v can be found in Kang and Park (2014).

4. Case study

The Western Shandong power grid of China is taken as an example to demonstrate the effectiveness of the method. There are 168 large substations (220 kV and above) and 213 transmission lines (220 kV and above) in this system. Therefore, the nodes which are not needed in this step is temporarily neglected to explicitly demonstrate the subsystem, namely only important load-center substations, corresponding key transmission lines (220 kV and above) and large thermal power plants are reserved. According to restoration plan of Shandong Power Grid, it is assumed that at the beginning of backbone-network reconfiguration, the whole system is divided into two subsystems to accelerate the restoration process. The two subsystems are separated with a dotted line in Fig. 2. In subsystem 1, unit 5 of Shihengyi power plant has been restored successfully by Taishan pumped storage power plant. In subsystem 2, three power plants including Xindian, Zhangdian and Yixi have been restored to establish the early network. The following step is to determine the next restoration schemes.

The restoration processes are similar because the two subsystems are at the same stage of restoration. The decision-making process of subsystem 1 is introduced in this case in detail, and the restoration of subsystem 2 proceeds in a similar way. According to the flowchart in Fig. 1, at the beginning of decision-making, the restoration goals and restoration paths need to be determined. For subsystem 1, it is in the first stage of backbone-network reconfiguration, large units are key targets. After the restoration targets are determined, path searching method is implemented for establishing the restoration paths. The candidate restoration schemes can be given in Table 4. After the restoration schemes are determined, security checking is implemented. The following candidate restoration schemes satisfy security demands of restoration, and no adjustments are needed.

After the candidate restoration schemes are determined, the evaluation index system and values of attributes need to be determined. Attributes for evaluation include switching overvoltage f_1, transient voltage dip caused by startup of large motors f_2, sustained power frequency overvoltage f_3, restoration duration f_4, importance of units f_5 and restoration risk f_6. For switching overvoltage, it is expressed by $A_{so} = (U_{allow} - U_{actual})/U_{allow}$, where U_{allow} is the maximum allowable switching overvoltage, U_{actual} is the maximum actual switching overvoltage in the worst situation. For transient voltage dip, $A_{vdip} = (U_{\min} - U_{set})/U_{set}$, where $U_{\min}$ is the minimum voltage, U_{set} is the setting value of low voltage protection. For sustained power frequency overvoltage, it is expressed by $A_{spfo} = (U_{allowpf} - U_{actualpf})/U_{allowpf}$, where $U_{allowpf}$ is the maximum allowable sustained power frequency overvoltage, $U_{actualpf}$ is the actual maximum sustained power frequency overvoltage. Restoration duration is based on the startup time of units and operating time of corresponding equipment. Many factors including states of units, limitation of critical time, type of units, capacity of units, and startup time of auxiliary motors are considered. Precise restoration time cannot be determined, only the range of restoration duration can be given. Importance of units is expressed by linguistic terms considering capacity of units, influence on consequent restoration, geological distribution of units, and priority of load. Restoration risk is comprehensively decided by factors including but not limited to voltage levels, reliability of units, length of transmission lines and number of switch operation. In different restoration stages of different power systems, different factors need to be considered. The decisions should be made according to the real structure and status of the system, and the linguistic terms need to be adjusted according to the experiences and knowledge of experts. Some factors can be ignored in some restoration stages and in some power systems. For example, in the late stage of restoration, overvoltage is not a major concern because of adequate measures for controlling the voltage. Even for evaluating the same attributes in different systems, different factors need to be counted. Some factors are important in some

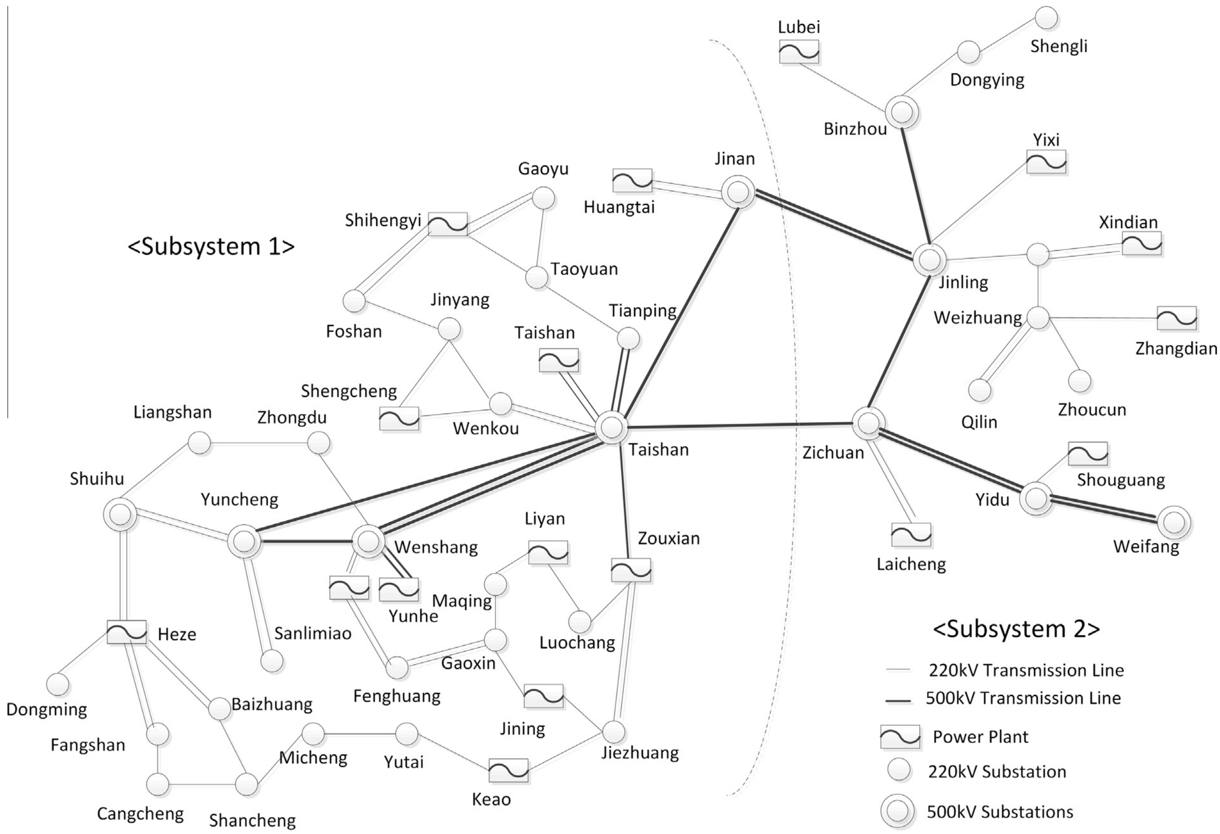

Fig. 2. Structure of the Western Shandong power grid of China.

Table 4
Candidate restoration schemes.

	Restoration schemes
1	Taishan substation → Yuncheng Substation → Shuihu Substation → Unit 5 of Heze power plant
2	Taishan substation → Unit 3 of Zouxian power plant
3	Taishan substation → Jinan substation → Unit 5 of Huangtai power plant
4	Taishan substation → Wenkou substation → Unit 1 of Shengcheng power plant
5	Taishan substation → Wenshang substation → Unit 5 of Yunhe power plant

systems, but negligible in other systems. There are no immutable evaluation index systems which can be adapted to all kinds of decision situations in power system restoration. The evaluation values of candidate restoration schemes are shown in Table 5.

The Euclidean distance is calculated after linguistic terms are translated to fuzzy numbers in this case. After data processing, the normalized decision matrix is as follows:

$$\begin{bmatrix} 0.09211 & 0.01087 & 0.4068 & [0.5385, 0.6087] & [0.6, 0.7, 0.8] & [0.2, 0.3, 0.4] \\ 0.7368 & 1.000 & 0.9322 & [0.7778, 0.9333] & [0.8, 0.9, 1.0] & [0.6, 0.7, 0.8] \\ 0.1184 & 0.8261 & 1.000 & [0.6087, 0.7000] & [0.8, 0.9, 1.0] & [0.4, 0.5, 0.6] \\ 1.000 & 0.6033 & 0.6949 & [0.8750, 1.000] & [0.4, 0.5, 0.6] & [0.8, 0.9, 1.0] \\ 0.1184 & 0.4293 & 0.4068 & [0.6364, 0.7000] & [0.6, 0.7, 0.8] & [0.4, 0.5, 0.6] \end{bmatrix}$$

Without loss of generality, supposing there are two decision makers, they measure the relative importance of attributes respectively with multiplicative preference relations C_1 and fuzzy preference relations C_2. If more decision makers participate in the restoration and give other forms of preference relations, different

Table 5
Attribute values of candidate restoration schemes.

Restoration scheme	f_1/p.u.	f_2/p.u.	f_3/p.u.	f_4/min	f_5	f_6
1	1.93	0.652	1.076	[230 260]	MH	ML
2	1.44	0.834	1.045	[150 180]	H	MH
3	1.91	0.802	1.041	[200 230]	H	M
4	1.86	0.761	1.059	[140 160]	M	H
5	1.91	0.729	1.076	[200 220]	MH	M

preference relations can be transformed into multiplicative preference relations. The processing method is introduced in Section 3.3.1.

$$\boldsymbol{C}_1 = \begin{bmatrix} 1 & 2 & 1/2 & 1/7 & 1/8 & 1/7 \\ 1/2 & 1 & 1/3 & 1/8 & 1/9 & 1/8 \\ 2 & 3 & 1 & 1/5 & 1/7 & 1/6 \\ 7 & 8 & 5 & 1 & 1/2 & 1/2 \\ 8 & 9 & 7 & 2 & 1 & 2 \\ 7 & 8 & 6 & 2 & 1/2 & 1 \end{bmatrix}, \quad \boldsymbol{C}_2 = \begin{bmatrix} 0.5 & 0.6 & 0.4 & 0.2 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.1 & 0.1 & 0.1 \\ 0.6 & 0.6 & 0.5 & 0.3 & 0.2 & 0.2 \\ 0.8 & 0.9 & 0.7 & 0.5 & 0.4 & 0.2 \\ 0.9 & 0.9 & 0.8 & 0.6 & 0.5 & 0.4 \\ 0.8 & 0.9 & 0.8 & 0.6 & 0.4 & 0.5 \end{bmatrix}$$

The weights of DMs can be obtained by AHP method (Wang & Parkan, 2006), negotiations or expert experiences. The weights of the two decision makers are set 0.3 and 0.7, respectively as an example, according to their past performance, experiences and reliability. The subjective weights can be obtained using (9), and $w' = (0.0440, 0.0352, 0.0593, 0.2200, 0.3643, 0.2772)$ $(p = 2)$. The objective weights are obtained from the normalized decision matrix, $w'' = (0.2468, 0.2408, 0.1736, 0.0954, 0.1014, 0.1420)$. Combined weights are integrated using (17), $w = (0.1251, 0.1105, 0.1218, 0.1738, 0.2307, 0.2381)$. Corresponding values of S_i, R_i, Q_i are calculated by the abovementioned algorithm, and they are tabulated in Table 6.

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Table 6
Evaluation values of candidate restoration schemes.

	1	2	3	4	5
S	0.1491	0.0281	0.0746	0.0472	0.1182
R	0.0389	0.0130	0.0300	0.0251	0.0300
Q	1.000	0.000	0.5241	0.3132	0.7015

Table 7
Stability intervals and sensitivity analysis results.

Attributes	Initial weights	Stability interval	Sensitivity
f_1	0.1251	[0, 0.2926]	7.993605
f_2	0.1105	[0, 0.2725]	9.049774
f_3	0.1218	[0, 0.3039]	8.210181
f_4	0.1738	[0, 0.5663]	5.75374
f_5	0.2307	[0.1106, 0.2882]	17.3913
f_6	0.2381	[0, 0.3942]	6.40615

According to the sequence of Q_i, the compromise solution is a_2, and it satisfies the requirements of VIKOR method. The optimal sequence is (2,4,3,5,1). The highest-priority of a_2 is because Zouxian power plant is very important in Shandong power grid, field experiment has been done for a few times, the dispatchers are very familiar with this plant. The 3×60 MVar shunt reactors installed in Zouxian can help control overvoltage and voltage stability of the system. The high-priority of a_4 is because of its smaller capacity, shorter transmission lines, better safety margin and less restoration risk. It is advantageous to restore Shengcheng power plant to keep the network stable at the early and delicate stage of restoration. For the subjective preferences on the importance of units, a_3 is preferred. Huangtai power plant is responsible for the power supply of Jinan, the provincial capital of Shandong province. The political and economic significance is prominent. Unit 5 of Heze power plant is not preferable because its energization needs a long transmission line, the restoration speed is unsatisfactory. The most important is that the transient voltage dip is close to its lowest limitation, it is easy to violate the limitations for some uncertain factors.

Stability intervals and sensitivity analysis results are tabulated in Table 7.

The sensitivity of restoration duration is the highest. Therefore in the evaluation, more attention should be paid on this attribute, the relative importance between restoration duration and other attributes need to be focused on.

Continue the above-mentioned process for next restoration target until the whole network is restored, and produce the restoration plan with these restoration steps. For subsystem 2, a similar restoration process can be put forward, the optimal restoration sequence is (Zibo substation $\rightarrow$ Zichuan substation $\rightarrow$ Laicheng power plant). After the two subsystems are restored respectively, they are synchronized to reconstruct the power system.

5. Conclusions

The paper extends the implementation of MADM from the stage of black-start to backbone-network reconfiguration. Considering the deficiencies of SAW in real applications, VIKOR method is utilized for restoration to comprehensively deal with both qualitative and quantitative criteria. The VIKOR method for hybrid attributes can reflect the fuzziness and uncertainties in practical restoration problems, and avoid too much fuzzification. Forcing DMs to provide preference relations in a specific form leads to errors sometimes, an integrated mathematical programming model is established to obtain consistent subjective preferences. VIKOR can reduce restoration risks for considering not only the overall performance of all criteria but also the most unsatisfactory attributes. Delay and even failure of restoration caused by quantification or transformation of attributes is decreased, transformation errors are eliminated and information losses of data processing are reduced. Sensitivity of attributes' weights is defined and analyzed to guide the DMs. Performance results of Shandong power grid of China demonstrate the applicability and credibility of the method.

Despite all the advantages of the hybrid VIKOR method, there are still some limitations. First, sensitivity analysis in this paper may need too much time. Second, the number of alternatives may be very big for security checking. At last, other types of attributes like intuitionistic fuzzy numbers are not considered in this paper.

The developed method has a promising perspective for restoration. It can be utilized after complete blackouts as well as local outages. Future work will focus on determining candidate restoration schemes, promoting generalization performance of restoration strategies, proposing quick and accurate restoration path searching methods and speeding up security checking. For the MADM method, there are also some works, such as simplifying decision-making process by screening, simplifying the weighting method and sensitivity analysis, and establishing a dynamic evaluation index system.

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